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AN INVESTIGATION OF RADIANT HEAT TRANSFER IN ABSORBING, EMITTING AND SCATTERING MEDIA

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FOREWORD

The work reported herein was performed by the University of Oklahoma Research Institute, Norman, Oklahoma under Air Force Contract No. AF 33(657) - 8859, Project 7116, Task 70192, sponsored by the Thermo-Mechanics Branch, Aeronautical Research Laboratories, Office of Aerospace Research, United States Air Force.

The study was conceived by the author during the tenure of a National Science Foundation Science Faculty Fellowship at Purdue University in 1960-61 and serves as the basis for a thesis submitted to the Faculty of Purdue University in partial fulfillment of the Degree of Doctor of Philosophy.

The guidance of Professor R. J. Grosh, Head of Mechanical Engineering, Purdue University, Professor C. M. Sliepcevich, Associate Dean, College of Engineering, University of Oklahoma, and other members of the Graduate Committee is gratefully acknowledged.

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Special acknowledgment should also be accorded Mr. Paul W. Schreiber, Monitoring Scientist, for his interest, suggestions and critical reviews during the progress of the project.

This is an interim technical report for this project initiated in April, 1962 and completed in December, 1962. All interim reports were informal letter reports of progress.

ABSTRACT

This is a study designed to examine the effect of anisotropic scattering on radiant heat transfer. The equation of transfer for a scattering, absorbing, and emitting medium is simplified by restricting the analysis to axially symmetric, plane parallel geometry. The scattering media are assumed composed of spherical particles of uniform diameter and complex refractive index. The scattering functions are taken from the literature and are the result of application of the Mie theory of electromagnetic scattering.

The following problems are considered:

1. Radiant heat transfer between a diffuse plane wall and an isothermal, scattering, absorbing, and emitting media of infinite extent.

2. Radiant heat transfer between two diffusely reflecting and emitting plane walls and an isothermal separating medium which scatters, absorbs and emits radiant energy.

3. Radiant heat transfer between two plane walls separated by an absorbing, emitting, and scattering medium in radiative equilibrium.

4. Radiant heat transfer to a diffuse surface bounding one side of an absorbing and scattering medium of finite optical thickness with a uniform parallel flux incident in a normal direction on the "free surface" of the medium.

Results are presented for scattering by spheres composed of material having refractive indices of 1.25 - 1.25i and 2.00 - 0.60i, corresponding roughly to iron and carbon respectively, and the case of isotropic scattering. The results include the effect of variations of surface reflectivity, optical spacing, and particle size to wave length ratio.

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

Many heat transfer problems involve radiant energy transmission through media having local inhomogeneities which effect the transmission. Some specific examples of such problems include the following: The microscopic carbon particles in a luminous flame, the fibers in fibrous thermal insulation, the particles in fluid bed reactors and heat exchangers, the pigment particles in paint, the particles and fibers in re-entry heat shields, the luminous particles in solid propellant rocket exhaust.

In each case these inhomogeneities absorb, emit, reflect, and transmit varying percentages of the radiant energy. The combination of reflection and transmission is called radiation scattering. For a rigorous treatment of problems involving scattering, both the direction and intensity of the scattered radiation must be accounted for. The scattering of electromagnetic energy from single particles has been studied extensively in the optical range in certain branches of astrophysics, colloid chemistry, and meteorology. In his book "Light Scattering by Small Particles" Van De Hulst (34) gives a review of work beginning with Tyndall who noted the scattering of blue light by smoke particles, continuing through the works of Rayleigh, Gans and Mie, recounting in considerable detail the results of modern investigators applying these analyses. Stratton (31) gives a detailed development of the Mie solution to Maxwell's Equation for the scattering by a sphere with an incident plane parallel radiation, and forms the basis for the extensive analyses by many investigators in colloid chemistry and meteorology. A rather extensive bibliography is presented by Fishman (12) of the work in light scattering up to 1957.

Problems in heat transfer involve not only the scattering of radiant energy from a single particle, but must account for the scattering of a diffuse radiant field by very large numbers of particles which in turn absorb and scatter radiant energy originating or scattered from other particles. Such a problem is called a problem in multiple scattering.

A survey of the literature indicates that most researchers in heat transfer have neglected the effects of scattering in their analysis. Jakob (19) reviews the early works of Shack, Nusselt and Wohlenberg who have independently studied luminous flames and the combustion of powdered coal. More recent studies include the works presented in proceedings of the 1961 International Heat Transfer Conference (28,33,37). These papers present the radiation effects in an empirical manner utilizing a measured emissivity without regard for effects of multiple scattering. However, it should be noted that one paper in the above group, the work of Thring, Foster, McGrath, and Ashton, (33) of University of Sheffield, does utilize Mie theory in the determination of an extinction cross section of the particles which will account for effects of size variation and refractive index. Multiple scattering effects were neglected, however, in investigations for large optical depths.

Radiant transmission through porous and fiborous insulating materials has been treated by a number of investigators. Jakob (18) reviews the early work of Nusselt which introduced an apparent radiative thermal conductivity proportional to the cube of the absolute temperature and a spacing length. Verschoor and Greebler (35) introduced an opacity factor in a similar expression which must be determined by spectroscopic

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techniques. Hamaker (15) utilized a "two flux" transport equation originally developed by Schuster (30) to investigate the effects of multiple scattering. Larkin (22) extended this method by accounting for the difference in forward and backward scattering. Glaser (13) investigating radiant transfer in evacuated powder insulations derives an expression for apparent radiative conductivity similar to Verschoor and Greebler. However, the opacity factor is derived on the basis of light scattering theory. He discusses qualitatively the effects of optimum particle size and the use of foil radiation shields. Both Larkin and Glaser indicate that Mie theory for spheres may be used for determination of a scattering function for powder or porous insulations. A note of caution should be injected here since closely spaced spheres will not scatter radiation independently and the anticipated effects of partizle size are probably erroneous. This will be discussed in more detail later in this report.

None of the above investigators have accounted quantitatively for the effects of diffusely reflecting and emitting walls bounding the insulation space. These effects will form an important feature of this investigation.

Fluidized beds of particles represent an area of interest in many industries. When high temperatures are involved, radiant transport may become a significant mode of heat transfer. Hawkins and co-workers (16) considered radiation to clouds of particles from a wall but neglected the scattering effects. In considering the clouds of particles as purely absorbing, the extinction cross section is effectively taken equal to the absorption cross section which is assumed identical to the geometric cross section of the particles. The computation for total radiation then involved a finite difference consideration of layers of the cloud. Hill and Wilhelm (17) have studied radiant heat transfer in quiescent beds of particles utilizing a finite difference technique which divided the medium into imaginary discrete surfaces which reflect, absorb and transmit radiant energy. The method does not lend itself to calculations from the basic physical properties of the medium, however. Zabrodsky (38) recognizes the contribution of radiant transfer in fluidized beds for high temperatures. The expression he uses for a radiative coefficient in an approximate form and does not consider the scattering of radiation by the particles.

Paint may be considered as a dispersion of particles in a semi transparent vehicle. Although the radiative transfer of usual interest is primarily in the optical range, the basic problem of multiple scattering is the same as that treated in this work. Blevin and Brown (2) utilized an analysis involving an approximate method developed by Chandrasekhar (4) for a semi infinite dispersion. However, the influence of a bounding reflecting surface was not considered. This problem is similar to the problem of radiant transfer to a re-enforced plastic "heat shield" used for re-entry vehicles.

Investigators utilizing a more extensive treatment of the influence of scattering than the two flux method for the problem of heat transfer include Viskanta (36). He presents a rather extensive bibliography of the problem of radiative transfer in absorbing and scattering media. The equation of transfer is derived in a general form. However, the only problem solved involving scattering is the case of radiative equilibrium with isotropic scattering. Incidently, the calculation for the met radiant flux in this problem becomes identical to the calculation for the radiant flux in a purely absorbing media if the extinction coefficient is used in place of the absorption coefficient in computing the optical depth. Adrianov (1) presented an electrical analog for the solution of the problem of radiant transfer with absorption emission and isotropic scattering in a diffuse enclosure reflecting and emitting plane walls.

A literature search indicated that there has been no study of radiant heat transfer where anisotropic scattering has been considered.

As mentioned earlier, most of the work in the area of multiple scattering has been accomplished in the areas of astrophysics, colloid chemistry and meterology. By far the most extensive work in the area is "Radiative Transfer" by Chandrasekhar (4). As an astrophysicist he is primarily interested in two problems, (a) the reflection of planetary atmospheres and (b) radiation from stellar atmospheres. In each case, monochromatic intensities as a function of the angle of exit from the atmosphere are of primary importance. In all cases the atmospheres are considered to be semi infinite in depth. Therefore reflections at surfaces and the total net flux are not considered.

Problems in Meteorology and Colloid Chemistry usually involve particle size determinations and transmission and reflection characteristics. Most of the work in this area has been based on single scattering theory. However, Chu and Churchill (6) published a numerical method of analyzing the reflection and transmission of a plane cloud of particles. More recently Churchill and co-workers (8) published a report of an exact solution for the same problem based on a set of integral equations for the reflectivity and transmissivity of plane dispersions derived by Chandrasekhar (4). Churchill (8) also develops a modified diffusion approximation which he demonstrates as being very close to the exact representation for small optical thicknesses. (The elementary diffusion approximation has been shown valid for larger optical depth by Roseland.) It should be noted that this method does not provide for reflecting or diffusely emitting boundary conditions. In addition the exact solution is limited to scattering functions which may be represented by a Legendre series of only a few terms. A practical limit of two terms is used in this paper.

It should be noted that the problem of neutron scattering may be considered analogous to radiative transfer in a scattering media. Davison (10) discusses various methods of solution of the neutron transport equation including the method of "discrete ordinates" developed by Chandrasekhar and utilized in a modified form in the present investigation. However, the boundary conditions and mechanism of neutron production are somewhat different in the neutron flux problems than in problems of heat transfer.

Putman, Kim and Neuhold (28) have recently written a digital computer solution to plane parallel radiative transfer which has been published as Aeronautical Systems Division Report No. ASD-TDR-62-544. This was written both for neutron transport problems and the problem of light transmission through the atmosphere. The method represents the Boltzman Equation as a finite difference equation and solves the problem as a network in x, θ, and φ space. The method is flexible and could be applied to heat transfer. However, for the purposes of this study, the method developed in this report is more useful in that it may be programmed on small computers and it gives a continuous solution in optical depth.

The present work is undertaken in view of the wide range of interest in the problem of radiation heat transfer through media with anisotropic scattering, and the absence of published information with regard to the effect of scattering in problems involving radiative transfer. It is the purpose of this investigation to develop a method of analysis which will provide a basis for the evaluation of the effect of scattering in problems of engineering interest. In particular the analysis will provide a method for computing the net radiative heat transfer between parallel diffusely

2. EXTINCTION OF RADIANT ENERGY: ABSORPTION AND SCATTERING

The measurement of the intensity of a ray of electromagnetic energy will indicate that the intensity decreases with distance as a pencil of rays passes through certain media. This decrease in intensity is called the extinction of the ray. This extinction is the result of two phenomena called absorption and scattering.

Absorption will occur when some of the photons of the pencil of rays are "trapped" by matter in the ray path. The energy of the trapped photons is then transformed into a form of internal energy of the matter. Some of this energy may be re-emitted as photons with a different frequency. This is sometimes called incoherent scattering. This re-emission will be considered as part of the scattering.

If the media through which the ray is passing has a discontinuous refractive index, some of the photons will be deflected from their original direction. This phenomena is called scattering and includes such effects as reflection, refraction and diffraction.

In the formulation of the radiant transport equation, the coefficients of extinction, scattering and absorption must be defined. Consider a single pencil of rays traversing an otherwise darkened medium. Let:

$I(x)$ = The monochromatic intensity of radiation of the ray in the direction of the ray and at some distance x along the path of the ray.

Then the change of intensity along the path of the ray may be expressed as follows:

$$\frac{dI(x)}{dx} = -\rho\beta I(x)$$

where:

ρ = mass of the scattering matter per unit volume of mixture

β = monochromatic mass extinction of coefficient

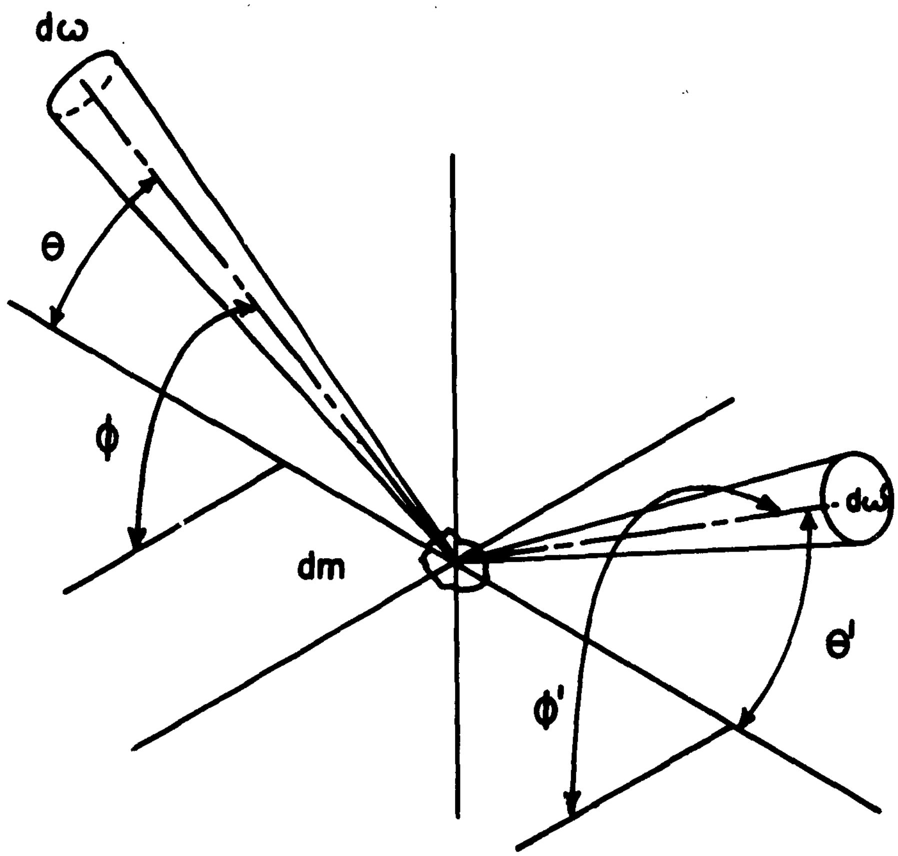
Since the extinction results from a combination of absorption and scattering, it is convenient to further define:

σ = the monochromatic mass scattering coefficient

$\varkappa$ = the monochromatic mass absorption coefficient

such that $\beta = (\sigma + \varkappa)$.

The energy which is scattered from a ray will contribute to the intensity of radiation in another direction. In order to consider the overall effects of scattering it is necessary to account for the direction of scattering. In general the scattering function $S(\theta,\varphi;\theta',\varphi')$ may be defined by the following expression which is illustrated in Figure (1). (Chandrasekhar (4) calls this a phase function and Van De Hulst refers to it as an amplitude function)



COORDINATE SCHEME FOR SCATTERING FUNCTION

FIGURE 1

$I(\theta,\varphi)S(\theta,\varphi;\theta'\varphi')\frac{d\omega'}{4\pi}\,d\nu d\omega dm$ = rate at which energy is scattered by the element of mass dm from a pencil of rays of intensity $I(\theta,\varphi)$ having a direction θ,φ with respect to an arbitrary reference direction, solid angled $d\omega$, and with a frequency interval $d\nu$, into a solid angle $d\omega'$ in a direction θ' and at an azimuthal angle φ' with respect to the reference directions. Borrowing the terminology of fluid mechanics this might be considered as an Eulerian approach.

Considering the problem from the so-called Lagrangian point of view $S(\theta,\varphi,\theta',\varphi')\frac{d\omega'}{4\pi}$ is a probability function expressing the probability that a photon in the frequency interval $d\nu$ incident on the element of mass dm, from a direction contained in the solid angle $d\omega$, characterized by the angles θ and φ from the co-ordinate axis, will be deflected into a direction within the solid angle $d\omega'$, characterized by the angles θ' and φ'. It can be readily seen that if the probability of the photon being "trapped" or deflected from its original direction as it encounters the elemental mass is β, and the probability that the photon is trapped is expressed as κ, then:

$$\int_0^{4\pi} S(\theta,\varphi;\theta',\varphi')\frac{d\omega'}{4\pi} = \sigma$$

where σ is the probability that the photon will be scattered, that is deflected from its original direction.

In this work we shall assume that the scattered photons have the same frequency (energy) as the incident photon. It is assumed that the energy of the trapped photon is entirely converted into thermal energy of the mass element. Photons having a spectral energy distribution corresponding to the Planck function at the temperature of the elemental mass are assumed to be contributed to the radiation field in an isotropic manner. The probability of a photon within the frequency interval $d\nu$ being emitted in a direction contained in the solid angle $d\omega$ is here assumed to be $\kappa I_{bb}(T)$ where: $I_{bb}(T)$ = the value of the Planck function at the frequency ν and temperature T of the element of mass dm. The ratio $\frac{\sigma}{\beta}$, which will be important in the subsequent analysis, has been given the name "albedo of single scattering" in the problems of radiative transfer in astrophysics.

It can be seen that the characteristics of this function will depend upon the character of the local inhomogenieties in the refractive index. In most cases of practical interest in heat transfer calculations these inhomogenieties may be considered to be particles of solid or liquid suspended in a transparent medium. The shape, size and relative refractive index all have a significant effect on the scattering. If the particles are closely spaced, (separation of less than three diameters as estimated by Van De Hulst) the scattering may also depend upon the relative position and spacing of the particles. In this case the scattering is said to be dependent. Because of the added independent variables and geometric complexities, theoretical determination of the phase function has not been made for dependent light scattering. (Of course, in the study of crystal structure with high energy radiation, dependent scattering is of prime importance.)

The scattering by a single particle would be called single independent scattering. It can be seen that if a relatively few particles exist within the elemental volume, the measured intensity at some distance away from the volume will have the character of scattering by a single particle but will have an amplitude

proportional to the number of particles in the volume. Such scattering is also called single scattering. It can be seen that as the number of particles is increased each particle is subjected to increased radiation scattered by other particles. Thus, the resulting character of the scattered energy is influenced by the multiple scattering.

It can be seen that the effects of multiple scattering can be theoretically determined by summing the effects of extinction and single scattering by all of the particles in the region.

The determination of the scattering function has been the object of much interest in recent years. Van De Hulst (33) reviews in some detail the work prior to 1957.

In summary, however, the theoretical determination is made by solving Maxwell's Equations for the propagation of electromagnetic waves with the appropriate boundary conditions. The problem was first solved by Gustav Mie in 1908 for spheres. The mathematical and physical background for the Mie solutions has been presented by Stratton (30) and others. Values of the computed perpendicularly polarized components of light scattered by spheres may be found tabulated in the literature (3, 5,7,11,14,24,26,27,33). Most of the work has been done for real refractive indices, since this serves the purpose of many of the problems of chemistry and astrophysics.

However, the real refractive index applies strictly only to particles of perfect dielectric material. As demonstrated by Stratton (30) and other standard texts on electromagnetic theory, the electrical conductivity of a material gives rise to an imaginary component in the refractive index. This in effect results in a dampening of the electromagnetic wave according to Maxwell's theory. Thus, part of the energy of the wave transmitted into the particle is converted into another form of energy, in other words absorbed within the particle. In the case of the real refractive index the incident wave energy is conserved and equals the total of the energy leaving the particle.

In many situations the particles may be of odd shapes and sizes with the values of the complex refractive index often unknown over much of the spectrum. The analytical determination of the scattering function as well as the extinction coefficient for actual particles may thus become virtually impossible. It is therefore desirable that a method for experimental determination of the scattering be developed. Such a method will be discussed in Chapter 6.

In this study, the system of primary concern is an axially symmetric system. This requires that the scattering function be axially symmetric and that the conditions of incident intensities and reflectivities be uniform and axially symmetric over the infinite plane boundaries of the system.

An axially symmetric scattering function is here defined as describing the condition in which the intensity of the radiation scattered from a pencil of rays incident on an elemental volume is axially symmetric with respect to the direction of the incident pencil. A further requirement is that the scattered intensity be independent of the direction of the incident pencil with respect to the axis of the system.

An example of a particle having an axially symmetric scattering function would be a sphere whose composition varies at most with its radius. Practical application of the method can be extended, however, to systems of particles with odd shapes and

sizes by considering an experimentally determined "average" scattering function for randomly oriented and spaced particles. In such a case the particle size and spacing must remain very small compared to the characteristic dimension of the system.

Thus for the case of a particle with an axially symmetric scattering function, the function may be tabulated as a function of a single angle. (Usually the angle between the scattered ray and the forward direction of the incident pencil of rays.)

Most of the published results of computations of scattering functions are for real refractive indices. This represents the case of conservative scattering in which absorption of the radiant energy is neglected. In all actual cases the refractive index is a complex number with the imaginary component resulting from the absorption. The inclusion of the complex refractive index materially increases the computation necessary to obtain the scattering function. For most purposes for which scattering information has been used, the ideal case neglecting the imaginary form has been used.

Because of the unavailability of scattering functions for the nonconservative case, the computations in this study have been made assuming a normalized scattering function $S(\theta,\varphi;\theta',\varphi')$ equal to the scattering function computed from the real component of the refractive index. The values of σ and β have been taken from tabulated values resulting from computations which take into account the complex nature of the refractive index. Churchill, Chu, etc. (8) have utilized this same method of approximating the scattering function.

3. THE EQUATION OF TRANSFER

In selecting a method for the solution of radiation heat transfer through clouds of particles involving the multiple scattering effects, there are two possible types of approaches. The first possibility might be a Lagrangian approach utilizing statistical probability theory. This has been used successfully in some types of neutron transport problems, and is called a Monte Carlo Method. In essence, the method "traces" a large number of photons individually, utilizing probability theory for free path determination and scattering direction. By accounting for a large number of randomly selected photons in the system an overall "average" system behavior may be determined. In practice, large computer storage and long operating times are usually required for suitable solutions.

The second method which might be termed an Eulerian approach involves writing a radiant energy balance about an elemental volume taken along a pencil of rays enclosed within an incremental solid angle. The equation expressing such a balance has been called the equation of transfer in problems of radiative transfer and is analogous to the Boltzman transport equation in neutron transport theory. This second approach has been selected for this analysis.

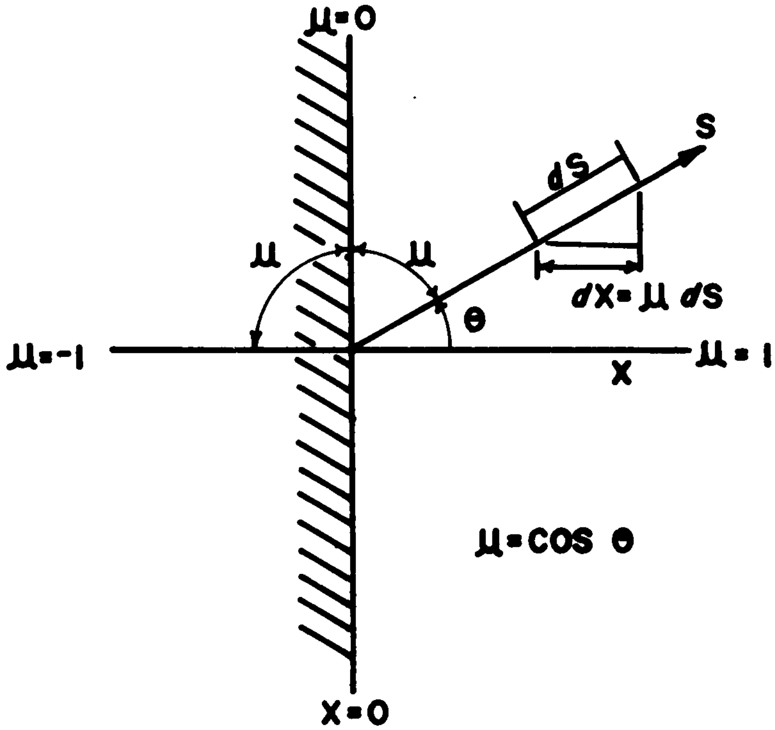
The derivation of the equation of transfer may be found in numerous works, (4, 20,35) and a general derivation will not be repeated here. For the purpose of clarity in presentation, however, the special form of the equation will be derived adaptable to the specific problems of this report.

Let s represent distance measured along a pencil of rays penetrating a medium consisting of a uniform dispersion of particles suspended in a non-absorbing substance with a refractive index of unity. The quantity ρ represents the mass of scattering particles per unit volume of the medium. The equation for conservation of radiant energy as the pencil of rays traverses an elemental volume of the media containing a mass of scattering particles dm may be written:

$$\frac{dI(s,\theta,\varphi)}{ds} = -\rho\beta I(s,\theta,\varphi) + \frac{\rho\sigma}{4\pi}\int_0^{\pi}\int_0^{2\pi} S(\theta,\varphi;\theta',\varphi')I(s,\theta'\varphi')\sin\theta' d\varphi' d\theta' + \rho\kappa I_{bb}(s) \quad (1)$$

where the left side of the equation represents the rate of decrease of intensity along the pencil of rays. The first term on the right represents the extinction of the ray in accordance with the definition of the mass extinction coefficient. The second term on the right represents the contribution of intensity in the direction of (θ,φ) by radiant energy scattered into that direction by radiation traversing the elemental volume from all directions. It should be noted that the scattering function utilized here is the normalized function and the factor σ represents an average effect over all directions and is thus not an exact expression for real cases. The third term in the right side of the equation represents the contribution to the intensity by emission from the elemental volume. $I_{bb}(s)$ representing the Planck function corresponding to the frequency interval for which the equation is written and κ, the monochromatic mass absorption coefficient which is assumed equal to the emission coefficient for the purposes of this investigation.

For the case of a plane parallel system it is convenient to express the distance s in terms of the distance measured along the normal to the bounding surface (Figure 2). Denoting this distance as x and letting the positive direction of x be the reference



COORDINATE SCHEME FOR AXIALLY SYMMETRIC CASE

FIGURE 2

direction then:

$$s = \frac{x}{\mu} \tag{2}$$

where: $\mu = \cos\theta$

θ = the angle between the forward direction of the pencil of rays and the reference direction

Upon dividing the equation by $\rho\beta$ and by application of the noted transformation of variables the equation of transfer becomes:

$$\mu \frac{dI(x,\mu,\varphi)}{\rho\beta dx} = -I(x,\mu,\varphi) + \frac{\sigma}{4\pi\beta}\int_{-1}^{\pi}\int_{0}^{2\pi} s(\mu,\varphi;\mu'\varphi')I(x,\mu',\varphi')d\varphi' d\mu' + \frac{\kappa}{\beta} I_{bb}(x) \tag{3}$$

It is convenient to express distance in terms of optical spacings. Therefore let τ represent optical depth, where τ is defined by the following equation:

$$\tau = \int_{0}^{x} \rho\beta dx \tag{4}$$

We now restrict the analysis to the case of an axially symmetric system. This restriction implies a plane parallel system with uniform boundary conditions which are axially symmetric with respect to the normal direction of reference . The scattering function of the medium must also be axially symmetric. The intensity thus becomes independent of the azimuthal angle and the equation of transfer may be written

$$\mu \frac{dI(\tau,\mu)}{d\tau} = -I(\tau,\mu) + \frac{\sigma}{4\pi\beta}\int_{-1}^{+1} I(\tau,\mu')\int_{0}^{2\pi} S(\theta)d\varphi' d\mu' + \frac{\kappa}{\beta} I_{bb}(\tau) \tag{5}$$

where $S(\theta)$ is the axially symmetric scattering function of the media. From solid geometry:

$$\cos\theta = \mu\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\varphi - \varphi')$$

Letting: $$S(\mu,\mu') = \frac{1}{2\pi}\int_{0}^{2\pi} S(\theta)d\varphi' \tag{6}$$

the equation of transfer is then written:

$$\mu \frac{dI(\tau,\mu)}{d\mu} = -I(\tau,\mu) + \frac{\sigma}{2\beta}\int_{-1}^{+1} I(\tau,\mu')S(\mu,\mu')d\mu' + \frac{\kappa}{\beta} I_{bb}(\tau) \tag{7}$$

This integro-differential equation for intensity is thus an exact representation for the case of an axially symmetric system. It should be noted, however, that although the scattering must result from finite inhomogeneities within the media, the equation is derived assuming a continuum. Some restriction must therefore be placed on the size and spacing of the particles with respect to the characteristic dimensions of the system. This restriction is necessary because scattering functions and extinction coefficients normally give the resultant effect at a large distance from the particle. A safe estimate is that the particle size should be at least two orders of magnitude smaller than the thickness of the system.

4. SOLUTIONS OF THE EQUATION OF TRANSFER

4.1 SELECTION OF A QUADRATURE FORMULA

The equation of transfer (7) for a plane parallel axially symmetric system has been presented in the previous chapter. Exact solutions for this integro-differential equation will of course depend on the form of the scattering function. Chandrasekhar (4) has shown that a method which utilizes a numerical quadrature formula may be shown to approach the exact solution for certain bounding conditions and with certain relatively simple scattering functions. For actual cases of engineering analysis, where the scattering function may be known only through experimental techniques and the boundary conditions usually include partially reflecting surfaces, such an exact analysis is extremely complicated. However, the use of a quadrature formula such as Chandrasekhar uses to develop his exact solution should serve as an excellent method for approximate computations which are entirely suitable for engineering purposes.

Such a method simplifies to some extent the representation of experimental values of the scattering function by requiring the function to be known only at discrete positions. In addition, with reflecting boundaries, the integrals representing the radiosity of the surfaces are conveniently represented by the quadrature formula.

The equation of transfer is written as follows utilizing the quadrature formula:

$$\mu_i \frac{dI(\tau,\mu_i)}{d\tau} = -I(\tau,\mu_i) + \frac{\sigma}{2\beta}\sum_{j=1}^{n} a_j S(\mu_i,\mu_j) I(\tau,\mu_j) + \frac{\kappa}{\beta} I_{bb}(\tau) \tag{12}$$

where: μ_i, μ_j represent the discrete ordinates specified by quadrature formula.

a_j is the weight factor corresponding to the ordinate specified by the formula.

The integro-differential equation is now represented by n ordinary first order differential equations which are to be solved simultaneously. It is desirable to select n as small as possible and still maintain a suitable degree of representation.

The Gaussian Formula for quadrature is suggested by Chandrasekhar. In this formula the weight factor is based on the zeros of the Legendre Polynomial $P_n(\mu)$, and

$$a_j = \frac{1}{P'_n(\mu_j)} \int_{-1}^{+1} \frac{P_n(\mu)}{\mu-\mu_j}\, d\mu$$

This method will give an exact value for the integral is $S(\mu_i,\mu_j)I(\tau,\mu_j)$ can be represented by a polynomial in μ of order 2n-1 or less. It thus would be expected to have an accuracy similar to a numerical quadrature having 2n-1 equally spaced ordinates. Thus, the use of the Gaussian Formula will substantially reduce the number of equations required for a given degree of accuracy. This advantage easily offsets the disadvantage of the irregular intervals and weight factors required by the quadrature.

Sykes (31) has pointed to the discontinuity at $\mu = 0$ experienced at the

boundaries in problems of this type. He suggests expressing the equations in terms of components such that $0 < \mu \leq 1$ and $-1 \leq \mu < 0$. This requires a variation of the Gaussian Formula such that:

$$\int_0^1 S(\mu,\mu')I(\tau,\mu')d\mu' = \sum_{j=1}^{n} a_j S(\mu_i,\mu_j)I(\tau,\mu_j) \tag{14}$$

The a_j, μ_j, μ_i correspond to the weight factors and specified ordinates of the so called double Gaussian Formula. The equation of transfer can then be written as the following set of equations:

$$+\mu_i \frac{dI(\tau, +\mu_i)}{d\tau} = -I(\tau, +\mu_i) + \frac{\sigma}{2\beta}\sum_{j=1}^{n} a_j S(+\mu_i, +\mu_j)I(\tau, +\mu_j) + \frac{\sigma}{2\beta}\sum_{j=1}^{n} a_j S(+\mu_i,-\mu_j) I(\tau, -\mu_j) + \frac{\kappa}{\beta} I_{bb}(\tau) \tag{15}$$

$$-\mu_i \frac{dI(\tau, -\mu_i)}{d\tau} = -I(\tau, -\mu_i) + \frac{\sigma}{2\beta}\sum_{j=1}^{n} a_j S(-\mu_i, +\mu_j)I(\tau, +\mu_j) + \frac{\sigma}{2\beta}\sum_{j=1}^{n} a_j S(-\mu_i,-\mu_j) I(\tau, -\mu_j) + \frac{\kappa}{\beta} I_{bb}(\tau)$$

The equation of transfer thus becomes 2n simultaneous, linear, non-homogeneous first order differential equations. The methods for obtaining a solution for the homogeneous set of equations is well known. It is noted that the last term in the equations is the non-homogeneous term and involves the monochromatic intensity of black body radiation which is a complicated function of temperature at the optical depth τ. In the case of a constant temperature medium, the non-homogeneous term becomes constant and the solution of the equation is somewhat simpler than for complicated temperature profiles. It should be possible, however, to develop solutions for more complex profiles. In this work only the case of the constant temperature media and the case of radiative equilibrium will be considered.

The case of a constant temperature media actually approximates many real situations in which the particles move rapidly past the wall in a turbulent fashion such that the temperature gradient at the wall is very steep with respect to optical depth.

Consider first the solution of the set of corresponding homogeneous equations. A solution of the following form is assumed and substituted into the set of equations.

$$I(\tau, +\mu_i) = x_i e^{\gamma\tau} \qquad I(\tau, -\mu_i) = x_{(i+n)} e^{\gamma\tau} \tag{16}$$

Upon substitution of the assumed solution into the equations, a system of homogeneous algebraic equations is obtained. The resulting set of simultaneous algebraic equations may be expressed in matrix form as follows:

Let: $$b_{p,q} = \frac{\sigma}{2\beta}\left(\frac{a_q S_{p,q}}{\mu_p}\right) \tag{17}$$

where $a_q = a_j \qquad j = q \qquad q = 1, 2, \ldots, n$

$a_q = a_j \qquad j = q-n \qquad q = n+1, n+2, \ldots, 2n$

$\mu_p = \mu_j \qquad j = p \qquad p = 1, 2, \ldots, n$

$\mu_p = \mu_j \qquad j = p-n \qquad p = n+1, n+2, \ldots, 2n$

(Recall that a_j is the weight factor and μ_j is the ordinate $j = 1, 2, \ldots, n$ specified by the quadrature formula)

$$S_{p,q} = S(+\mu_i, +\mu_j) \qquad \begin{array}{l} i = p \\ j = q \end{array} \qquad \begin{array}{l} p = 1, 2, \ldots, n \\ q = 1, 2, \ldots, n \end{array}$$

$$S_{p,q} = S(-\mu_i, -\mu_j) = S(+\mu_i, +\mu_j) \qquad \begin{array}{l} i = p-n \\ j = 2-n \end{array} \qquad \begin{array}{l} p = n+1, n+2, \ldots, 2n \\ q = n+1, n+2, \ldots, 2n \end{array}$$

$$S_{p,q} = S(+\mu_i, -\mu_j) = S(-\mu_i, +\mu_j) \qquad \begin{array}{l} i = p \\ j = q-n \end{array} \qquad \begin{array}{l} p = 1, 2, \ldots, n \\ q = n+1, n+2, \ldots, 2n \end{array}$$

$$S_{p,q} = S(-\mu_i, +\mu_j) = S(+\mu_i, -\mu_j) \qquad \begin{array}{l} i = p-n \\ j = q \end{array} \qquad \begin{array}{l} p = n+1, n+2, \ldots, 2n \\ q = 1, 2, \ldots, n \end{array}$$

The set of equations may then be represented in matrix form as follows:

$$\left[b_{p,q} \right] \left[x_p \right] - \left[\frac{\delta_{pq}}{\mu_p} \right] \left[x_p \right] - \gamma \left[x_p \right] = 0 \qquad (18)$$

Where: δ_{pq} = Kroneker Delta

$\delta_{pq} = 1 \qquad p = q$

$\delta_{pq} = 0 \qquad p \neq q$

For a non-trivial solution to this set of equations the following matrix must be equated to zero.

$$\left[b_{pq} - \left(\frac{1}{\mu_p} + \gamma \right) \delta_{pq} \right] = 0 \qquad (19)$$

The system of equations may be solved by numerical techniques. The 2n possible values for γ are called the characteristic values or eigenvalues of the matrix. Associated with each eigenvalue, there will be 2n values of x_i, $x_{(i+n)}$ $i = 1, \ldots, n$ called the eigenvectors. Since there are only 2n equations these values are obviously determined only as ratios, and a constant of integration must be determined by application of the boundary conditions. The solution for the homogeneous set of equations then becomes:

$$I(\tau, +\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau} \qquad I(\tau, -\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau} \tag{20}$$

The particular solution associated with the constant temperature atmosphere is readily found by assuming the solution equal to a constant and substituting into the differential equation. Let $I_p = C$

Then $$0 = -C + \frac{\sigma}{2\beta} \sum_{j=1}^{n} \left[a_j S(+\mu_i, +\mu_j)C + a_j S(+\mu_i, -\mu_j) \right] - \frac{\varkappa}{\beta} I_{bb}(T_a) \tag{21}$$

Factoring C and noting that:

$$\sum_{j=1}^{n} \left[a_j S(\mu_i, \mu_j) + a_j S(\mu_i, -\mu_j) \right] = 2 \quad \text{gives:} \quad C \frac{(\beta-\sigma)}{\beta} = \frac{\varkappa}{\beta} I_{bb}(T_a) \tag{22}$$

Since by definition $\beta-\sigma = \varkappa$, then $C = I_{bb}(T_a)$ (23)

This agrees with intuitive reasoning.

The solution for the radiant transport equation in the plane parallel, axially symmetric, constant temperature media thus becomes:

$$I(\tau, +\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a) \qquad I(\tau, -\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a) \tag{24}$$

Where only the intensities in the discrete directions corresponding to the ordinates of the quadrature formula are specified the 2n constants C_α remain to be evaluated by the application of the bounding conditions of the problem.

4.2 THE PROBLEM WITH PARALLEL DIFFUSE WALLS

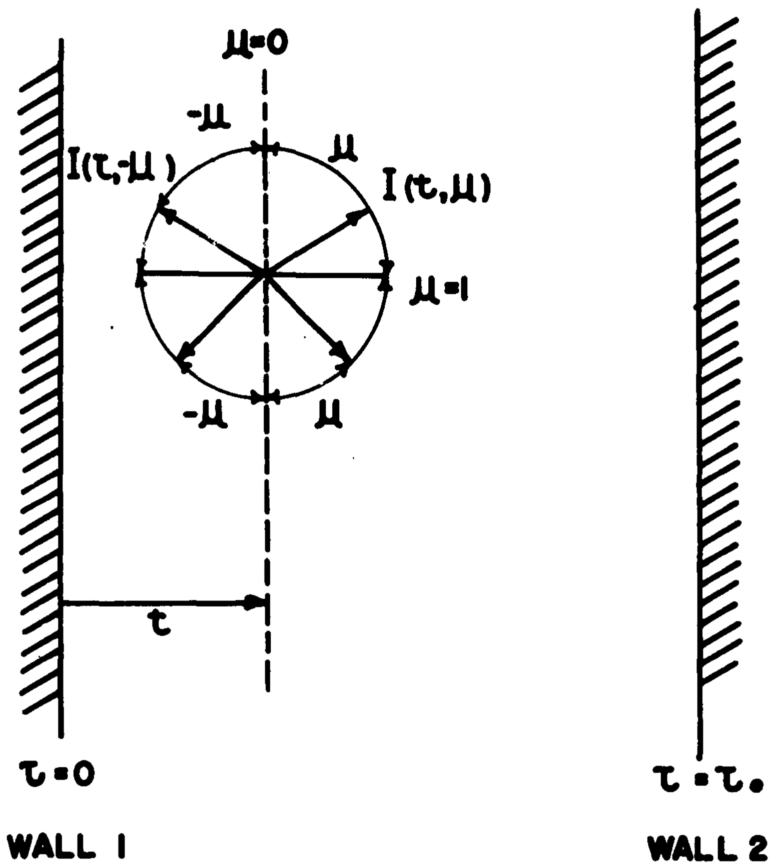
Consider the solution of the equation of transfer in a constant temperature plane cloud of finite thickness, bounded by parallel diffusely emitting and reflecting walls, having constant temperatures T_1 and T_2 respectively (Figure 3).

The monochromatic radiosity R of a surface is defined as the sum of the reflected and emitted radiation leaving the surface. The restriction that the walls be diffuse requires that the intensity of the radiant energy leaving the surface be independent of direction. Thus the 2n boundary conditions needed to evaluate may be written:

$$I(0, +\mu_i) = \frac{R_1}{\pi} \qquad I(\tau, -\mu_i) = \frac{R_2}{\pi}$$

where: τ = The optical spacing of the walls
R_1, R_2 = Monochromatic radiosity of Wall 1 and Wall 2 respectively

From the definition of radiosity, the following relationships may be written:



COORDINATE SCHEME FOR PROBLEM WITH PARALLEL WALLS

FIGURE 3

$$R_1 = \epsilon_1 \pi I_{bb}(T_1) + 2\pi\rho_1 \int_{-1}^{0} \mu\, I(0,\mu)\, d\mu$$

$$R_2 = \epsilon_2 \pi I_{bb}(T_2) + 2\pi\rho_2 \int_{0}^{1} \mu\, I(\tau_o,\mu)\, d\mu \tag{26}$$

where: $\epsilon_1\ \epsilon_2$ = The monochromatic emissivities of surface 1 and 2 respectively

$\rho_1\ \rho_2$ = The monochromatic reflectivities of surface 1 and 2 respectively

$I_{bb}(T_1), I_{bb}(T_2)$ = The monochromatic black body intensity corresponding to the temperatures of surface 1 and 2.

Noting that $\epsilon = (1 - \rho)$ and again utilizing the quadrature formula to replace the integral gives:

$$R_1 = (1-\rho_1)\,\pi\, I_{bb}(T_1) + 2\pi\rho_1 \sum_{j=1}^{n} a_j\mu_j\, I(0, -\mu_j)$$

$$R_2 = (1-\rho_2)\pi\, I_{bb}(T_2) + 2\pi\rho_2 \sum_{j=1}^{n} a_j\mu_j\, I(\tau_o, +\mu_j) \tag{27}$$

Now utilizing equations (24) and (25), the above expression may be rewritten:

$$\sum_{\alpha=1}^{2n} C_\alpha x_{1,\alpha} + I_{bb}(T_a) = (1-\rho_1)\, I_{bb}(T_1) + 2\rho_1 \sum_{j=1}^{n}\sum_{\alpha=1}^{2n} a_j\mu_j C_\alpha x_{(j+n),\alpha} + \sum_{j=1}^{n} a_j\mu_j I_{bb}(T_a)$$

$$\sum_{\alpha=1}^{2n} C_\alpha x_{(1+n),\alpha} e^{\gamma_\alpha \tau_o} + I_{bb}(T) = (1-\rho_2) I_{bb}(T_2) + 2\rho_2 \sum_{j=1}^{n}\sum_{\alpha=1}^{2n} a_j\mu_j C_\alpha x_{j,\alpha} e^{\gamma_\alpha \tau_o} + \sum_{j=1}^{n} a_j\mu_j I_{bb}(T_a) \tag{28}$$

Collecting terms and noting that:

$$2\sum_{j=1}^{n} a_j\mu_j = 1$$

gives:

$$\sum_{\alpha=1}^{2n} C_\alpha \left[x_{1,\alpha} - 2\rho_1 \sum_{j=1}^{n} a_j\mu_j x_{(j+n),\alpha}\right] = (1-\rho_1)\, I_{bb}(T_1) - (1-\rho_1)\, I_{bb}(T_a)$$

$$\sum_{\alpha=1}^{2n} C_\alpha \left[x_{(1+n),\alpha} e^{\gamma_\alpha \tau_o} - 2\rho_2 \sum_{j=1}^{n} a_j\mu_j x_{j,\alpha} e^{\gamma_\alpha \tau_o}\right] = (1-\rho_2) I_{bb}(T_2) + (1-\rho_2) I_{bb}(T_a) \tag{29}$$

This is a set of simultaneous algebraic equations in C_α and may be written in matrix notation as follows:

$$\left[a_{r,\alpha}\right]\left[C_{\alpha}\right] = \left[b_p\right] \tag{30}$$

where

$$a_{r,\alpha} = x_{i,\alpha} - 2\rho_1 \sum_{j=1}^{n} a_j \mu_j x_{(j+n),\alpha} \quad \text{for } r = i \quad i = 1,2, \ldots, n$$

$$a_{r,\alpha} = x_{(i+n),\alpha} e^{\gamma_\alpha \tau_o} - 2\rho_2 \sum_{j=1}^{n} a_j \mu_j x_{j,\alpha} e^{\gamma_\alpha \tau_o} \quad \text{for } r = i+n \quad i = 1, 2, \ldots, n \tag{31}$$

$$b_p = (1-\rho_1)\, I_{bb}\,(T_1) - (1 - \rho_1)\, I_{bb}\,(T_a) \qquad p = 1, 2, \ldots, n$$

$$b_p = (1-\rho_2)\, I_{bb}\,(T_2) - (1 - \rho_2)\, I_{bb}\,(T_a) \qquad p = n+1, n+2, \ldots, 2n \tag{32}$$

Thus pre-multiplying both sides of the equation by the inverse of $\left[a_{r,\alpha}\right]$ gives:

$$\left[C_{\alpha}\right] = \left[a_{r,\alpha}\right]^{-1}\left[b_p\right] \tag{33}$$

By retaining the expressions for $I_{bb}(T_1)$, $I_{bb}(T_a)$ during the matrix multiplication and collecting terms, the monochromatic intensity at any optical depth $\tau_1, o \leq \tau_o$ may be written as follows:

$$I(\tau_1,\mu_i) = D_i(\tau_1)I_{bb}(T_1) + E_i(\tau_1)I_{bb}(T_2) + F_i(\tau_1)I_{bb}(T_a)$$

$$I(\tau_1-\mu_i) = G_i(\tau_1)I_{bb}(T_1) + H_i(\tau_1)I_{bb}(T_2) + K_i(\tau_1)I_{bb}(T_a) \tag{34}$$

Where:

$$D_i(\tau_1) = (1-\rho_1)\sum_{\alpha=1}^{2n} L_{a,\alpha} x_{i,\alpha} e^{\gamma_\alpha \tau_1}$$

$$E_i(\tau_1) = (1-\rho_2)\sum_{\alpha=1}^{2n} L_{b,\alpha} x_{i,\alpha} e^{\gamma_\alpha \tau_1}$$

$$F_i(\tau_1) = 1 - \left[D_i(\tau_1) + E_i(\tau_1)\right]$$

$$G_i(\tau_1) = (1-\rho_1)\sum_{\alpha=1}^{2n} L_{a,\alpha} x_{(i+n)_\alpha} e^{\gamma_\alpha \tau_1}$$

$$H_i(\tau_1) = (1-\rho_2) \sum_{\alpha=1}^{2n} L_{b,\alpha} X(i+n)_\alpha e^{\gamma_\alpha \tau_1}$$

$$K_i(\tau_1) = 1 - \left[G_i(\tau_1) + H_i(\tau_1) \right]$$

Where $L_{a,\alpha}$ is the sum of the first n terms of the $\alpha^{\underline{th}}$ row of the matrix $\left[a_{r,\alpha} \right]^{-1}$

$L_{b,\alpha}$ is the sum of the n, n+1, ..., 2n terms of the $\alpha^{\underline{th}}$ row of the same matrix.

4.2.1 Determination of Monochromatic Flux

With the intensity, thus determined in the directions corresponding to the ordinates of the quadrature formula, the net monochromatic flux may be determined. By definition

$$(\tau_1) = 2\pi \int_{-1}^{+1} \mu \, I\, (\tau_1, \mu) \, d\mu \tag{35}$$

where (τ_1) = the net monochromatic flux

Utilizing the quadrature formula to express the integral gives:

$$(\tau_1) = 2\pi \sum_{j=1}^{n} a_i \mu_j I(\tau_1 + \mu_j) - \sum_{j=1}^{n} a_j \mu_j I(\tau_1, - \mu_j) \tag{35a}$$

Now substituting in equation (34) for the intensity at optical depth τ_1 gives

$$(\tau_1) = M\, I_{bb}(T_1) - N\, I_{bb}(T_2) - Q\, I_{bb}\,(T_a) \tag{36}$$

where:

$$M = 2\pi \sum_{j=1}^{n} a_j \mu_j \left[D_j(\tau_1) - G_j(\tau_1) \right]$$

$$N = -2\pi \sum_{j=1}^{n} a_j \mu_j \left[E_i(\tau_1) - H_i(\tau_1) \right]$$

$$Q = -2\pi \sum_{j=1}^{n} a_j \mu_j \left[F_i(\tau_1) - K_i(\tau_1) \right] = (M-N)$$

4.2.2 Integration of Monochromatic Flux

The method developed above permits the determination of the monochromatic flux only at discrete frequencies. Because of the complex manner in which the absorption and scattering depend on the wave length of the radiation it is impractical to attempt to develop a functional relationship which might be integrated over wave length. From a review of the nature of the scattering and absorption of electromagnetic radiation by particles it can be reasoned that the values of M, N, and Q will vary slowly with the variation in frequency. It is therefore desirable to utilize some method of numerical quadrature for the integration.

A Reiz (4) has constructed a quadrature based on the zero's of the Laquerre Polynomials such that:

$$\int_0^\infty e^{-x} f(x)dx = \sum_{j=1}^{n} a_j f(x_j) \tag{37}$$

where a_j's are the weight functions and the x_j's are the corresponding ordinates. The formula is exact if f(x) is a polynomial of order less than 2m. Thus, the formula in effect fits a polynomial of order 2m-1 through the points selected.

Consider the first term of the equation integrating the flux over all wave lengths.

$$q_{net}(\tau_1) = \int_0^\infty \left[M(\tau_1)I_{bb}(T_1) - N(\tau_1)I_{bb}(T_2) - Q(\tau_1)I_{bb}(T_a)\right] d\nu \tag{38}$$

Recall that:

$$I_{bb}(T_1) = \frac{2h\nu^3}{c^2} \frac{e^{-\frac{h\nu}{kT}}}{1-e^{-\frac{h\nu}{kT}}} \tag{39}$$

where:
- h = Planck's constant
- k = Boltzman's constant
- c = Velocity of light
- ν = The frequency of the radiation

Making this substitution gives:

$$\int_0 M(\tau_1)I_{bb}(T_1)\, d\nu = \int_0 e^{-\frac{h\nu}{kT_1}} \frac{2M(\tau_1)h\nu^3}{c^2 \left(1-e^{\frac{-h\nu}{kT_1}}\right)} d\nu \tag{40}$$

Let:

$$x = \frac{h\nu}{kT_1}$$

The equation becomes

$$\int_0^\infty M(\tau_1) I_{bb}(T_1)\, d\nu = \int_0^\infty e^{-x} \quad \frac{2M(x,\tau_1)\, k^4 T_1^4\, x^3}{h^3 c^2\, (1 - e^{-x})} \quad dx \tag{41}$$

It should be noted that $M(\tau_1)$ is dependent upon the radiation frequency and that the transformation of variable for the quantity $M(\tau_1)$ is noted $M(\tau_1) = M(x_1 \tau_1)$.

The equation (41) is now in the proper form for application of formula (37)

$$\int_0^\infty M(\tau_1) I_{bb}(T_1)\, d\nu = \sum_{j=1}^{m} a_j \frac{2M(x_j, \tau_1)\, k^4 T_1^4\, x_j^3}{h^3 c^2\, (1-e^{-x_j})} \tag{42}$$

The radiant heat transfer at the optical depth τ_1 can thus be represented as follows:

$$\begin{aligned} q_{net}(\tau_1) = {} & \frac{2k^4}{h^3 c^2} \sum_{j=1}^{m} \frac{a_j M(x_j, \tau_1)\, x_j^3}{(1-e^{-x_j})} \quad T_1^4 \quad + \\ & - \frac{2k^4}{h^3 c^2} \sum_{j=1}^{m} \frac{a_j N(x_j, \tau_1) x_j^3}{(1-e^{-x_j})} \quad T_2^4 \quad + \\ & - \frac{2k^4}{h^3 c^2} \sum_{j=1}^{m} \frac{a_j Q(x_j, \tau_1) x_j^3}{(1-e^{-x_j})} \quad T_a^4 \end{aligned} \tag{43}$$

It should be noted here that any computation of radiant heat transfer must begin with this expression since the frequencies at which M, N, and Q must be computed are determined by the temperatures of the surfaces and the atmosphere as well as the specified ordinates x_j of the quadrature formula.

The above computation is not restricted to gray surfaces since M, N, and Q are determined at discrete frequencies and the surface reflectivities at these frequencies will be utilized. The validity of the quadrature will be questionable, however, if the surface reflectivities vary sharply with wave length.

4.2.3 Limiting Case With Negligible Scattering

As the particle size in the cloud of particles becomes small compared to the incident radiation, the ratio of the scattering coefficient to the absorption coefficient $\frac{\sigma}{\beta}$ approaches zero. The integral thus disappears from the equation of

transfer and it may be written

$$\mu \frac{dI(\tau,\mu)}{d\tau} = -I(\tau,\mu) + I_{bb}(\tau)$$

This then is the equation of radiant transfer in an absorbing media such as a gas. Solutions for this equation with reflecting walls and a constant temperature media have appeared in the literature. Perhaps the most recent instance being a paper by Edwards. In this paper an "accounting" system was used to compute the effect of inter-reflections. The following solution appears to be somewhat simpler to use and to the author's knowledge has not appeared in previous publications.

For the case of parallel plates separated by a constant temperature media it is convenient to write the solution to the equation of transfer in discrete directions corresponding to the quadrature formula used in the previous problem.

$$\begin{aligned} I(\tau, +\mu_i) &= C_1 e^{-\frac{\tau}{\mu_i}} + I_{bb}(T_a) \\ I(\tau, -\mu_i) &= C_2 e^{-\frac{\tau-\tau_o}{\mu_i}} + I_{bb}(T_a) \end{aligned} \tag{45}$$

where C_1, C_2 are constants to be determined by the boundary conditions.

The boundary conditions again involve the integral equations associated with the boundary radiosities.

$$I(0, +\mu_i) = \frac{R_1}{\pi} = (1-\rho_1)I_{bb}(T_1) + 2\rho_1 \int_{-1}^{0} \mu\, I(0,\mu)d\mu$$

$$I(\tau_o, -\mu_i) = \frac{R_2}{\pi} = (1-\rho_2)I_{bb}(T_2) + 2\rho_2 \int_{0}^{1} \mu\, I(\tau_o,\mu)\, d\mu$$

As before the integral equation is solved by substitution of the quadrature formula for the integral. This gives:

$$\begin{aligned} I(0, +\mu_i) &= (1-\rho_1)I_{bb}(T_1) + 2\rho_1 \sum_{j=1}^{m} a_j\mu_j I(0,-\mu_j) \\ I(\tau_o,-\mu_i) &= (1-\rho_2)I_{bb}(T_2) + 2\rho_2 \sum_{j=1}^{m} a_j\mu_j I(\tau_o, +\mu_j) \end{aligned} \tag{47}$$

Substitution of (39) into (41) gives:

$$C_1 + I_{bb}(T_a) = (1-\rho_1)I_{bb}(T_1) + 2\rho_1 \sum_{j=1}^{n} a_j\mu_j \left[C_2 e^{-\frac{\tau_o}{\mu_j}} + I_{bb}(T_a)\right]$$

$$C_2 + I_{bb}(T_a) = (1-\rho_2)I_{bb}(T_2) + 2\rho_2 \sum_{j=1}^{n} a_j\mu_j \left[C_1 e^{-\frac{\tau_o}{\mu_j}} + I_{bb}(T_a)\right] \tag{48}$$

Let:

$$L = 2\sum_{j=1}^{n} a_j\mu_j e^{-\frac{\tau_o}{\mu_j}} \quad ,$$

and recall that: $2\sum_{j=1}^{n} a_j\mu_j = 1$

The equations (48) become: $C_1 - \rho_1 L C_2 = (1-\rho_1)I_{bb}(T_1) - (1-\rho_1)I_{bb}(T_a)$

$$-\rho_2 L C_1 + C_2 = (1-\rho_2)I_{bb}(T_2) - (1-\rho_2)I_{bb}(T_a) \tag{49}$$

Solving these equations for C_1, C_2 gives:

$$C_1 = \frac{1-\rho_1}{1-\rho_1\rho_2L^2} I_{bb}(T_1) + \frac{\rho_1 L(1-\rho_2)}{1-\rho_1\rho_2L^2} I_{bb}(T_2) - \frac{(1-\rho_1) + \rho_1 L(1-\rho_2)}{1-\rho_1\rho_2L^2} I_{bb}(T_a)$$

$$C_2 = \frac{\rho_2 L(1-\rho_1)}{1-\rho_1\rho_2L^2} I_{bb}(T_1) + \frac{(1-\rho_2)}{1-\rho_1\rho_2L^2} I_{bb}(T_2) - \frac{\rho_2 L(1-\rho_1) + (1-\rho_2)}{1-\rho_1\rho_2L^2} I_{bb}(T_a) \tag{50}$$

In order to compute the monochromatic flux, apply equation (36)

$$(\tau) = 2\pi \sum_{j=1}^{n} a_j\mu_j \left(C_1 e^{-\frac{\tau}{\mu_1}} - C_2 e^{\frac{\tau-\tau_o}{\mu_1}}\right) \tag{51}$$

In most instances the region of primary interest as far as heat transfer goes is at one of the surfaces. Considering the above equation at $\tau = 0$ gives:

$$(0) = 2\pi \sum_{j=1}^{n} a_j\mu_j \left[C_1 - C_2 e^{-\frac{\tau_o}{\mu_1}}\right] \tag{52}$$

$$(0) = \pi C_1 - \pi L C_2 \tag{53}$$

Substituting for C_1 and C_2 gives an equation of the following form:

$$(0) = M I_{bb}(T_1) - N I_{bb}(T_2) - Q I_{bb}(T_a) \tag{54}$$

where:

$$M = \pi \left[\frac{(1-\rho_1)(1-\rho_2 L^2)}{1-\rho_1\rho_2 L^2}\right] \tag{55}$$

$$N = \pi \left[\frac{L(1-\rho_2)(1-\rho_1)}{1-\rho_1\rho_2 L^2}\right] \tag{56}$$

$$Q = \left[M-N\right] \tag{57}$$

It can readily be seen that for the case of optically thin spacing:

$$\tau_o \simeq 0 \qquad L = 1$$

resulting in:

$$M = N = \frac{(1-\rho_1)(1-\rho_2)}{(1-\rho_1\rho_2)} \pi \; ; \; Q = 0 \tag{58}$$

The flux equation thus reduces to the elementary case of parallel gray walls.

In the case where $\tau_o \simeq \infty$, $L = 0$, it is readily seen that:

$$M \simeq \pi(1-\rho_1)$$

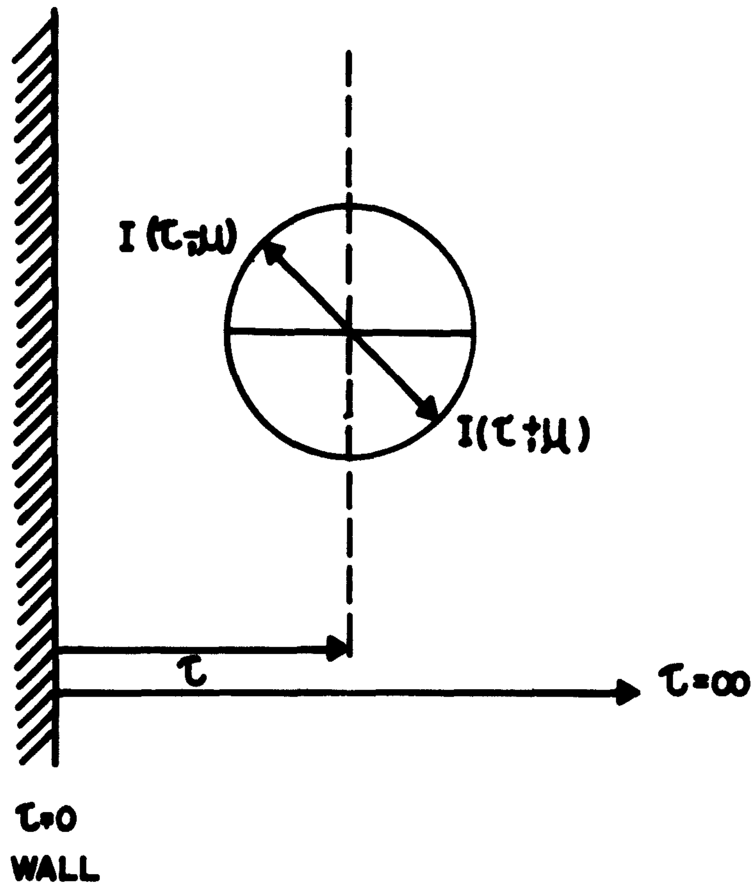
$$N \simeq 0$$

$$Q \simeq M$$

which again correspond to the elementary case.

4.3 THE RADIANT TRANSFER TO A SEMI-INFINITE CONSTANT TEMPERATURE ABSORBING AND SCATTERING MEDIA

In the case of a single infinite, diffuse, plane surface bounding a semi-infinite constant temperature absorbing, scattering, and emitting medium (Figure 4), the solution to the equation remains unchanged except for the constants of integration.



COORDINATE SCHEME FOR PROBLEM WITH SEMI-INFINITE MEDIUM

FIGURE 4

Actually this problem could be solved by allowing τ_o to become large in the case of the parallel surfaces. However, certain simplifying assumptions may be made which reduces the computation. In addition, such a computation should provide an additional limiting case for the problem of the parallel surfaces. Consider then, the general solutions as given before:

$$I(\tau, +\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a)$$

$$I(\tau, -\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a) \tag{24}$$

The boundary conditions are now

$$I(0, +\mu_i) = \frac{R_1}{\pi}$$

Limit

$$\tau \rightarrow \infty \qquad I(\tau, -\mu_i) = I(\tau, +\mu_i) = I_{bb}(T_a) \tag{59}$$

The second condition of (59) requires that $C_\alpha = 0$ for all α corresponding to $0 < \gamma_\alpha$.

It can be seen from the symmetry of the original set of simultaneous equations that n of the 2n eigenvalues will be negative and n will be positive. This leaves n values of γ_α to be determined by the n boundary conditions corresponding to the first equation (59). Let γ_α = negative eigenvalues ($\alpha = 1, 2, \ldots, n$), then:

$$a_{i,\alpha} = x_{i,\alpha} - 2\rho \sum_{j=1}^{n} a_j \mu_j x_{(j+n),\alpha} \tag{60}$$

represents the terms in the coefficient matrix of the set of simultaneous equations for C_α. Giving the equation

$$\left[a_{i,\alpha}\right] \left\lfloor C_\alpha \right\rfloor = \left\lfloor b_\alpha \right\rfloor \tag{61}$$

where b_α is a column matrix of equal terms such that

$$b_\alpha = (1-\rho_1) \left\{ I_{bb}(T_1) - I_{bb}(T_a) \right\} .$$

Multiplying both sides by the inverse, gives:

$$\left[C_\alpha \right] = \left[a_{i,\alpha} \right]^{-1} \left[b_\alpha \right] \tag{62}$$

Let:

$$D_i = \sum_{\alpha=1}^{n} C_\alpha x_{i,\alpha} e^{\nu_\alpha \tau}$$

$$F_i = \sum_{\alpha=1}^{n} C_\alpha x_{(i+n),\alpha} e^{\nu_\alpha \tau} \tag{63}$$

As in the previous problem the monochromatic flux is obtained by the integration of equation (35). Accomplishing this by use of the quadrature, the flux can then be written as follows:

$$(\tau) = M\, I_{bb}(T_w) - Q\, I_{bb}\,(T_a) \tag{64}$$

where:

$$M = 2\pi(1-\rho) \sum_{j=1}^{n} a_j \mu_j \left[D_j - F_j \right]$$
$$Q = M \tag{65}$$

This M may be compared directly with the corresponding value for the problem of parallel surfaces. The net radiant heat flux may be computed by integration over all wave length as in the previous case.

4.4 THE DETERMINATION OF RADIANT FLUX FOR THE CASE OF RADIATIVE EQUILIBRIUM

As a limiting case of certain real problems in which the conductive and convective modes of thermal energy transport are small, it is desirable to determine the radiant flux in a system in radiative equilibrium. Radiative equilibrium might be defined as a condition of steady state heat transfer through a medium which does not contain distributed heat sources or sinks. The sole mode of thermal energy exchange being radiant transfer. It is thus implied that the total radiant energy absorbed by an element of the medium must equal the total radiant energy emitted by the medium. The following equation is a result of this condition.

$$\int_0^\infty 2\pi\kappa \int_{-1}^{+1} I(\tau,\mu)\, d\mu d\nu = \int_0^\infty 4\pi\kappa\, I_{bb}(\tau)\, d\nu \tag{66}$$

This condition must hold for κ equal to any arbitrary function of ν , therefore,

the integrands must be identically equal.

$$\kappa \int_{-1}^{+1} I(\tau,\mu)\, d\mu = 2\kappa\, I_{bb}(\tau) \tag{67}$$

Utilizing the quadrature formula gives the expression

$$I_{bb}(\tau) = 1/2 \sum_{j=1}^{n} a_j I(\tau, +\mu_j) + \sum_{j=1}^{n} a_j I(\tau, -\mu_j) \tag{68}$$

This equation is substituted into the equation of transfer (15) giving:

$$\begin{aligned}
+\mu_i \frac{dI(\tau, +\mu_i)}{d\tau} &= - I(\tau, +\mu_i) + 1/2 \sum_{j=1}^{n} a_j\left[\frac{\sigma}{\beta} S(+\mu_i, +\mu_j) + \frac{\kappa}{\beta}\right] I(\tau, +\mu_j) + \\
&\quad + 1/2 \sum_{j=1}^{n} a_j \left[\frac{\sigma}{\beta} S(+\mu_i, -\mu_j) + \frac{\kappa}{\beta}\right] I(\tau, -\mu_j) \\
-\mu_i \frac{dI(\tau, -\mu_i)}{d\tau} &= -I(\tau, -\mu_i) + 1/2 \sum_{j=1}^{n} a_j\left[\frac{\sigma}{\beta} S(-\mu_i, +\mu_j) + \frac{\kappa}{\beta}\right] I(\tau, +\mu_j) + \\
&\quad + 1/2 \sum_{j=1}^{n} a_j\left[\frac{\sigma}{\beta} S(-\mu_i, -\mu_j) + \frac{\kappa}{\beta}\right] I(\tau, -\mu_j)
\end{aligned} \tag{69}$$

It is noted that these equations represent a set of linear, homogeneous, first order differential equations. The solution proceeds in a manner similar to the first problem discussed.

A solution is assumed to be of the form:

$$\begin{aligned}
I(\tau, +\mu_i) &= x_i e^{\gamma\tau} \\
I(\tau, -\mu_i) &= x_{(i+n)} e^{\gamma\tau}
\end{aligned} \tag{70}$$

Substitution into the equation of transfer gives a matrix equation identical to equation (18). However, the elements of the matrix $\left[\, b_{pq} \right]$ are now different. For this case equation (17) becomes:

$$b_{pq} = \frac{\sigma}{2\beta} \frac{a_q \delta_{pq}}{\mu_p} + \frac{\kappa a_q}{\beta} \tag{71}$$

Again there will be 2n eigenvalues and a set of 2n eigenvectors associated with each eigenvalue.

Since the differential equations are homogeneous there will be no particular solution and the equations for the intensity may be written:

$$I(\tau, +\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau}$$

$$I(\tau, -\mu_i) = \sum_{\alpha=1}^{2n} C_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau} \tag{72}$$

For the case of infinite plane, diffuse, surfaces separated by an absorbing, scattering and emitting medium in radiative equilibrium the boundary conditions may be expressed by equation (25) for the constant temperature medium. The determination of radiant flux proceeds in a manner similar to that of the constant temperature case, omitting the term containing $I_{bb}(T_a)$.

It should be noted that the results will be independent of τ, (but not of the optical spacing of the walls τ_o) since the physical situation implies a constant net flux. Thus in the resultant equations for the quantities corresponding to M, N,Q the following relationships should hold,

$$Q^e = 0$$

$$M^e = N \tag{73}$$

It should be noted, however, that M and N are dependent upon temperature corresponding to each wall and in general the equation for the monochromatic flux must be written:

$$\overline{\quad} = M^e\, I_{bb}(T_1) - N^e\, I_{bb}(T_2) \tag{74}$$

It is interesting to note that for isotropic scattering since $S(\mu_i, \pm \mu_j) = 1$, the equation of transfer becomes:

$$+\mu_i \frac{dI(\tau, +\mu_i)}{d\tau} = -I(\tau, +\mu_i) + 1/2 \sum_{j=1}^{n} a_j \left[I(\tau, +\mu_j) + I(\tau, -\mu_j) \right]$$

(75)

$$-\mu_i \frac{dI(\tau, -\mu_i)}{d\tau} = -I(\tau, -\mu_i) + 1/2 \sum_{j=1}^{n} a_j \left[I(\tau, +\mu_j) + I(\tau, -\mu_j) \right] \tag{75}$$

It can thus be seen that the problem for isotropic scattering in radiative equilibrium is identical to that in a purely absorbing and emitting media. This can easily be verified by noting that isotropic emission would be indistinguishable from isotropic scattering.

4.5 THE PROBLEM OF A NORMAL INCIDENT FLUX

The method utilized in the previous problem may also be adapted for the purpose of investigating the effect of certain types of protective coatings.

Consider a unidirectional flux normal to a plane parallel cloud of particles of finite optical thickness bounded by a diffuse surface on the side opposite the incident flux (Figure 5). It is assumed that the temperature of the particles is uniform. This case would approximate the effect of a pulse of radiant energy from a distant source, similar to that experienced in exposure to a nuclear blast.

The intensity of the unabsorbed, unscattered incident radiation will be accounted for separately from the scattered radiant flux.

Let I^o be the intensity of the incident flux and $d\omega$ be the solid angle subtended by the flux. Then $I^o e^{-\tau}$ will be the intensity of the attenuated flux at optical depth τ. The contribution to the scattered radiation field in the direction μ is $\sigma\, I^o\, d\omega e^{-\tau}\, \frac{S(\mu,1)}{4\pi}$ and the equation of transfer becomes:

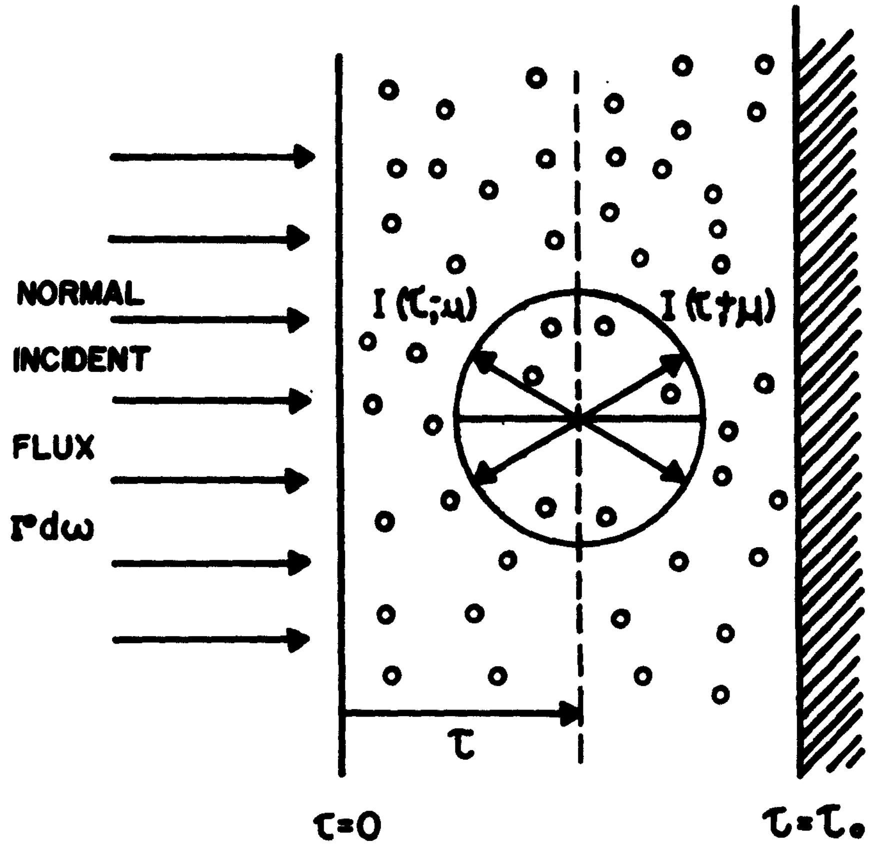
$$+\mu_i \frac{dI(\tau, +\mu_i)}{d\tau} = -I(\tau, +\mu_i) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j S(\mu_i, +\mu_j)\, I(\tau, +\mu_j) + $$

$$+ \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j S(\mu_i, -\mu_j)\, I(\tau, -\mu_j) + \frac{\sigma}{4\pi\beta}\, S(+\mu_i, 1)\, I^o d\omega e^{-\tau} + \frac{k}{\beta} I_{bb}(T_a)$$

$$\tag{76}$$

$$-\mu_i \frac{dI(\tau, -\mu_i)}{d\tau} = -I(\tau, -\mu_i) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j S(-\mu_i, +\mu_j)\, I(\tau, +\mu_j) + $$

$$+ \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j S(-\mu_i, -\mu_j)\, I(\tau, -\mu_j) + \frac{\sigma}{4\pi\beta}\, S(-\mu_i + 1)\, I^o d\omega e^{-\tau} + \frac{k}{\beta} I_{bb}(T_a)$$



COORDINATE SCHEME FOR PROBLEM
OF NORMAL INCIDENT FLUX

FIGURE 5

The homogeneous solution for this set of equations will be the same as equation (20). In order to find the particular solution associated with the scattered attenuated flux, the method of undetermined coefficients is used.

A solution of the following form is assumed.

$$I_p(\tau, +\mu_i) = \xi_i \left[\frac{\sigma}{4\pi\beta} S(+\mu_i, 1)\, I^o d\omega e^{-\tau} \right]$$
$$I_p(\tau, -\mu_i) = \xi_{(i+n)} \left[\frac{\sigma}{4\pi\beta} S(-\mu_i, 1)\, I^o d\omega e^{-\tau} \right] \tag{77}$$

where ξ is the coefficient which is to be determined by substitution into the equation of transfer. This gives the following set of simultaneous algebraic equations.

$$-\mu_i \xi_i S(+\mu_i, 1) = -\xi_i S(+\mu_i, 1) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} \xi_j a_j S(+\mu_i, +\mu_j)\, S(+\mu_j + 1) +$$
$$\frac{\sigma}{2\beta} \sum_{j=1}^{n} \xi_{(j+n)}\, a_j S(+\mu_i, -\mu_j)\, S(-\mu_j, 1) + S(+\mu_i, 1)$$
$$\mu_i \xi_{(i+n)} S(-\mu_i, 1) = -\xi_{1+n} S(-\mu_i, 1) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} \xi_j a_j S(-\mu_i, +\mu_j)\, S(+\mu_j, 1) +$$
$$+ \frac{\sigma}{2\beta} \sum_{j=1}^{n} \xi_{(j+n)}\, a_j S(-\mu_i, -\mu_j)\, S(-\mu_j, 1) + S(-\mu_i, 1) \tag{78}$$

This set of linear equations must now be solved for ξ_i. This set of equations may be written in matrix form as follows:

Let:

$$b_{pq} = \frac{\sigma\, a_q S_{pq} S_q}{2\beta} \tag{79}$$

where the subscripts have the same meaning as for equation (17).

$S_p, S_q = S(+\mu_i, 1)$ $\quad$ $i, j = p, q$ $\quad$ $p, q = 1, 2, \ldots, n$

$S_p, S_q = S(-\mu_i, 1)$ $\quad$ $i, j = (p-n), (q-n)$ $\quad$ $p, q = n+1, n+2, \ldots, 2n$

The matrix form of the equation is

$$\left[(S_p - \mu_p S_p)\, \delta_{pq} - b_{pq} \right] \left[\xi_p \right] = \left[\bar{S}_p \right] \tag{80}$$

Thus

$$\left\lfloor \xi_p \right\rfloor = \left\lfloor (S_p - \mu_p S_p)\, \delta_{pq} - b_{pq} \right\rfloor^{-1} \left[S_p \right] \tag{81}$$

The inversion may be performed by numerical techniques.

The equations for the intensities at the discrete directions may now be written as follows:

$$\begin{aligned} I(\tau, +\mu_i) &= \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a) + \varphi_i I^o\, d\omega e^{-\tau} \\ I(\tau, -\mu_i) &= \sum_{\alpha=1}^{2n} C_\alpha x_{i+n,\alpha} e^{\gamma_\alpha \tau} + I_{bb}(T_a) + \varphi_{i+n}\, I^o\, d\omega e^{-\tau} \end{aligned} \tag{82}$$

where:

$$\begin{aligned} \varphi_i &= \xi_i \left\lfloor \frac{\sigma}{4\pi\beta} S(+\mu_i, 1) \right] \\ \varphi_{i+n} &= \xi_{i+n} \left\lfloor \frac{\sigma}{4\pi\beta} S(-\mu_i, 1) \right) \end{aligned} \tag{83}$$

The constants C_α remain to be evaluated by the boundary conditions:

$$\begin{aligned} I(0, +\mu_i) &= 0 \\ I(\tau_o, -\mu_i) &= \frac{R}{\pi} \end{aligned} \tag{84}$$

R represents the radiosity of the diffuse surface bounding one side of the plane parallel dispersion of optical thickness τ_o.

$$R = (1-\rho)\, \pi\, I_{bb}(T) + 2\pi\rho \int_0^1 \mu\, I(\tau_o, \mu)\, d\mu + \rho e^{-\tau_o} I^o\, d\omega \tag{85}$$

where:

ρ = monochromatic reflectivity of the diffuse surface

T = temperature of the surface

Utilizing the quadrature formula to replace the integral as before, (84) is written

$$0 = \sum_{\alpha=1}^{2n} C_\alpha x_{i,\alpha} + I_{bb}(T_a) + \varphi_i\, I^o\, d\omega$$

$$\sum_{\alpha=1}^{2n} C_\alpha X_{(i+n),\alpha} e^{\gamma_\alpha \tau_o} + I_{bb}(T_a) + \varphi_{i+n} I^o d\omega e^{-\tau_o} =$$

$$(1-\rho) I_{bb}(T_1) + 2\rho \sum_{j=1}^{n} a_j \mu_j \left[\sum_{\alpha=1}^{2n} C_\alpha X_{i,\alpha} e^{\gamma_\alpha \tau_o} + I_{bb}(T_a) + \varphi_i I^o d\omega e^{-\tau_o} \right] + \tag{86}$$

$$+ \frac{\rho}{\pi} I^o d\omega e^{-\tau_o}$$

Rearrangement of these equations gives:

$$\sum_{\alpha=1}^{2n} C_\alpha X_{i,\alpha} = -I_{bb}(T_a) - \varphi_i I^o d\omega$$

$$\sum_{\alpha=1}^{2n} C_\alpha e^{\gamma_\alpha \tau_o} \left[X_{(i+n),\alpha} - 2\rho \sum_{j=1}^{n} a_j \mu_j X_{j,\alpha} \right] = \tag{86a}$$

$$(1-\rho)\left[I_{bb}(T) - I_{bb}(T_a) \right] + \left[2\rho \sum_{j=1}^{n} a_j \mu_j \varphi_j - \varphi_{i+n} + \frac{\rho}{\pi} \right] e^{-\tau_o} I^o d\omega$$

The C_α's may be determined utilizing matrix techniques.

Let $a_{p,\alpha}$ represent the terms of a matrix where

$$a_{p,\alpha} = X_{i,\alpha} \qquad p = 1, 2, \ldots, n$$

$$a_{p,\alpha} = \left[X_{(i+n),\alpha} - 2\rho \sum_{j=1}^{n} a_j \mu_j X_{j,\alpha} \right] e^{\gamma_\alpha \tau_o} \quad p = (i+n) = n+1, n+2, \ldots, 2n \tag{87}$$

Let

$$\left[B_\alpha \right] = \left[a_{p,\alpha} \right]^{-1} \left[b_p \right] \tag{88}$$

where $b_p = -1$ $\qquad p = 1, 2, \ldots, n$

$b_p = (\rho - 1)$ $\qquad p = n+1, n+2, \ldots, 2n$

and $\left[D_\alpha \right] = \left[a_{p,\alpha} \right]^{-1} \left[d_p \right]$ (89)

where $d_p = -\varphi_i$ $\quad p = 1, 2, \ldots, n = i$

$$d_p = \left[2\rho \sum_{j=1}^{n} a_j \mu_j \varphi_j + \frac{\rho}{\pi} - \varphi_p\right] e^{-\tau_o} \tag{90}$$

and $\left[E_\alpha\right] = \left[a_{p,\alpha}\right]^{-1} \left[e_p\right]$

where $e_p = 0$ $\quad p = 1, 2, \ldots, n$

$e_p = 1-\rho$ $\quad p = n+1, n+2, \ldots, 2n$

The equation for intensity may thus be written:

$$I(\tau,+\mu_i) = \left[\sum_{\alpha=1}^{2n} E_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau}\right] I_{bb}(T) + \left[1 + \sum_{\alpha=1}^{2n} B_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau}\right] I_{bb}(T_a) +$$

$$+ \left[\varphi_i e^{-\tau} + \sum_{\alpha=1}^{2n} D_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau}\right] I^o \, d\omega$$

$$I(\tau,-\mu_i) = \left[\sum_{\alpha=1}^{2n} E_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau}\right] I_{bb}(T) + \left[1 + \sum_{\alpha=1}^{2n} B_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau}\right] I_{bb}(T_a) + \tag{91}$$

$$+ \left[\varphi_{i+n} e^{-\tau} + \sum_{\alpha=1}^{2n} D_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau}\right] I^o \, d\omega$$

Again utilizing the quadrature formula to integrate the intensity over all directions in order to obtain the flux gives

$$= M^n I_{bb}(T) + N^n I_{bb}(T_a) + Q^n I^o \, d\omega \tag{92}$$

where:

$$M^n = 2\pi \sum_{j=1}^{n} a_j \mu_j \left[\sum_{\alpha=1}^{2n} E_\alpha e^{\gamma_\alpha \tau_o} (x_{i,\alpha} - x_{(i+n),\alpha})\right]$$

$$N^n = 2\pi \sum_{j=1}^{n} a_j \mu_j \left[\sum_{\alpha=1}^{2n} B_\alpha e^{\gamma_\alpha \tau_o} (x_{i,\alpha} - x_{(i+n),\alpha})\right] \tag{93}$$

$$Q^n = 2\pi \sum_{j=1}^{n} a_j \mu_j \left[\sum_{\alpha=1}^{2n} D_\alpha e^{\gamma_\alpha \tau_o}(x_{i,\alpha} - x_{(i+n),\alpha}) + (\varphi_i - \varphi_{i+n}) e^{-\tau_o}\right] + e^{-\tau_o}$$

The final term in the expression for Q^n is the result of the attenuated incident flux. The reflection of the incident flux is accounted for in the diffuse boundary conditions (85) for the radiosity at τ_0.

The integration over all frequencies of radiation may be accomplished in a manner similar to the previous problems.

For the case of isotropic scattering, the equations for ξ_i can be solved readily and an insight into the behavior of ξ_i may be gained. With the scattering functions taken as identically 1, the equations of transfer become

$$\mu_i \frac{dI(\tau, +\mu_i)}{d\tau} = -I(\tau, +\mu_i) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j \left[I(\tau, +\mu_j) + I(\tau, -\mu_j) \right] + \frac{\sigma}{4\pi\beta} I^o d\omega e^{-\tau} + \frac{\kappa}{\beta} I_{bb}(T_a)$$

$$-\mu_i \frac{dI(\tau, -\mu_i)}{d\tau} = -I(\tau, -\mu_i) + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j \; I(\tau, +\mu_j) + I(\tau, -\mu_j) + \frac{\sigma}{4\pi\beta} I^o d\omega e^{-\tau} + \frac{\kappa}{\beta} I_{bb}(T_a) \tag{94}$$

Assume the particular integral associated with the attenuated flux to be of the form:

$$I_p(\tau, +\mu_i) = \xi_i \left[\frac{\sigma}{4\pi\beta} I^o d\omega e^{-\tau} \right]$$

$$I_p(\tau, -\mu_i) = \xi_{i+n} \left[\frac{\sigma}{4\pi\beta} I^o d\omega e^{-\tau} \right]$$

Substituting this into the differential equations gives

$$-\xi_i \mu_i = -\xi_i + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j (\xi_j + \xi_{j+n}) + 1$$

$$\xi_{i+n} \mu_i = -\xi_{i+n} + \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j (\xi_j + \xi_{j+n}) + 1$$

Therefore,

$$\xi_i (1-\mu_i) = \xi_{i+n}(1+\mu_i)$$

Thus,

$$\xi_i (1-\mu_i) = \frac{\sigma}{2\beta} \sum_{j=1}^{n} a_j \xi_j \left[\frac{1}{1+\mu_j} \right] + 1$$

$$\xi_i = \frac{C}{1-\mu_i}$$

where C does not vary with the index i.

Solving for C gives:

$$C = \frac{1}{1-\frac{\sigma}{\beta}\sum_{j=1}^{n}\frac{a_j}{1-\mu_j^2}}$$

It is important to note that since $\frac{\sigma}{\beta}$ varies between 0 and 1 that if the summation in the denominator is greater than 1, (which it is for all orders of approximation) then C will have a singularity for some value of $\frac{\sigma}{\beta}$.

Chandrasekhar (4) on page 82 equation 90 of Radiative Transfer develops a similar expression for the case of isotropic scattering in a semi-infinite atmosphere with an incident flux. No mention of the singularity is made in Radiative Transfer although C (designated γ in radiative transfer) is utilized for all values of the albedo of single scattering $(\frac{\sigma}{\beta})$ in numerous cases.

The analysis, however, should not be influenced by this pole except for values of $\frac{\sigma}{\beta}$ which give large values of C as will be found in the neighborhood of the singularity. In this region numerical round off errors may be expected to give erratic results. The success of Chandrasekhar's analysis and the consistency of the results to be presented later in this report tend to verify this statement.

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5. NUMERICAL RESULTS

5.1 SELECTION OF SCATTERING FUNCTIONS

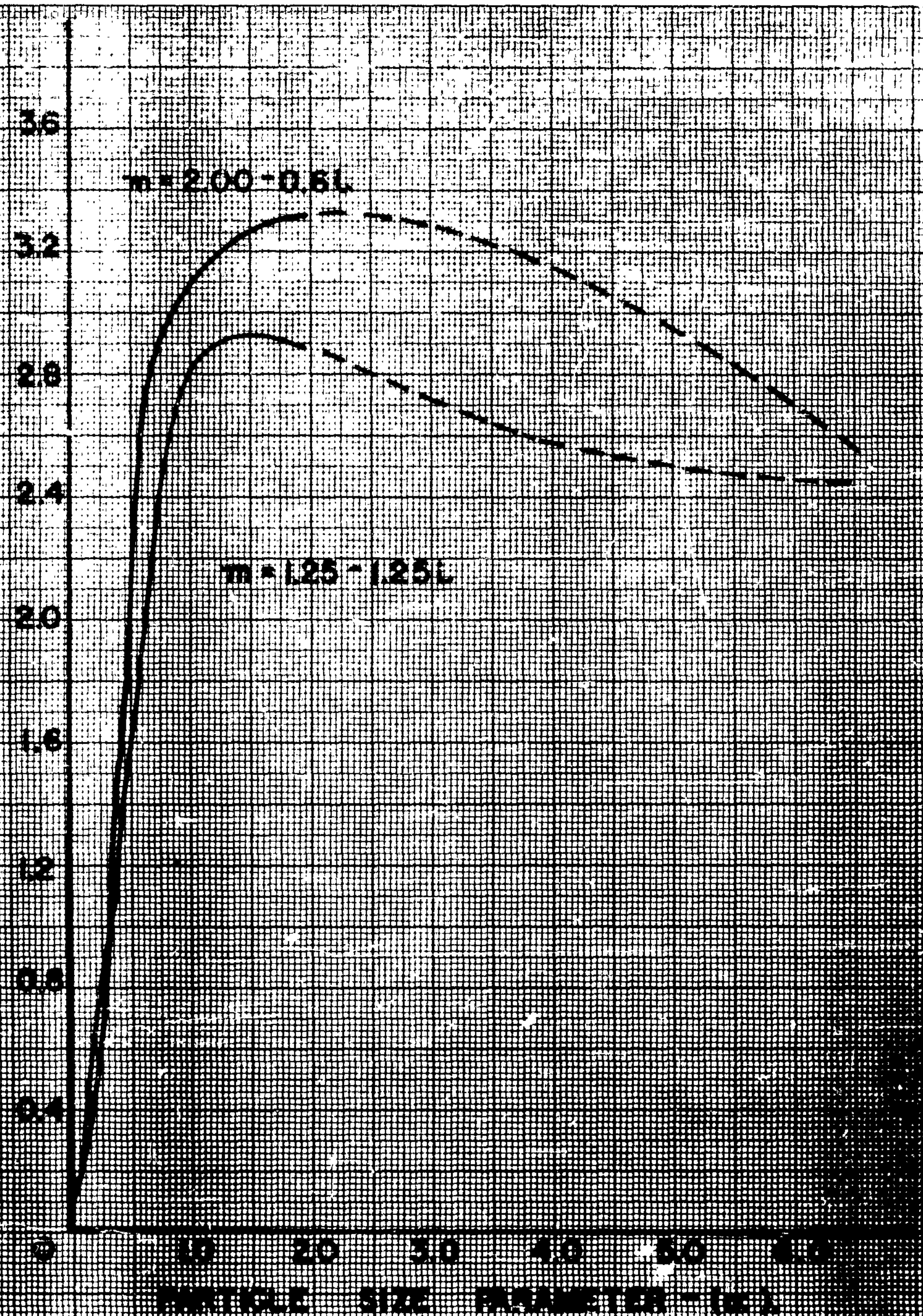
In order to perform the computations indicated in the previous chapter, the scattering function must be known. A considerable amount of work has been done in determining such functions for light scattering. Van De Hulst gives an extensive review of the work in this area prior to 1957. Fishman (12) has prepared an extensive bibliography on the subject.

For spherical particles of known refractive index, the scattering function may be computed based on the classical Mie solution of Maxwell's equation. The computations become very lengthy, however, for particles with complex refractive indices and for particles whose diameter is large with respect to the wave length of the illumination. Numerous tabulations have been published listing the results of large computer programs for certain refractive indices and particle size parameters. By far most of the work has been for particles with real refractive indices (i.e. particles which do not absorb energy). The reasons for this are (1) the computations for the real refractive index are much simpler than for the complex case (although the computations still are quite lengthy.) and (2) the real index approximation is valid for a large class of problems of interest in the fields of astrophysics, chemistry and meteorology.

In order to solve a problem of heat transfer utilizing analytically derived scattering functions, the investigator is faced with (1) the determination of the complex refractive index (since absorption and emission are important) of the material as a function of wave length or frequency (2) the determination of the scattering function over the range of particle size to wave length ratios. In both instances there is a scarcity of available data in the published literature, and extensive experimental and computational effort would be required for any given case.

For the purposes of this analysis, it was decided to use the work of Chu, Clark and Churchill (7) for the normalized angular distribution of intensity. These values were prepared for real refractive indices. The extinction and scattering coefficients used are taken from the work of Chromey (5) and are presented in Figures 6 and 7. For values of the particle size parameter above 2.0 the values were extrapolated toward the limiting values as given by Van De Hulst (33). For values of the size parameter less than 0.2, the ratio of scattering to extinction becomes very small and the extinction coefficient is computed by an approximation formula presented by Penndorf (26).

The justification for using this approximate method is based upon the unavailability of information of the angular distribution of intensity for the case of complex refractive indices. However, since the complex refractive index of most materials of engineering interest is unknown except at possibly a few optical frequencies, it is not considered feasible to expend the effort for the more exact scattering information at this time. In addition, the accuracy of the use of the approximate scattering function does not appear to be as bad as might be assumed. Deirmendjian (11) has demonstrated that the effect of the addition of an imaginary component to a real refractive index had the principal effect of "damping" out the oscillations of the intensity function plotted as a function of scattering angle. Because of the diffuse nature of the radiation field in the heat transfer problem, only the "average" effect of scattering is required and the effect of the numerical integration is to average



EXTINCTION CROSS SECTION K
AS A FUNCTION OF PARTICLE SIZE.

FIGURE 6

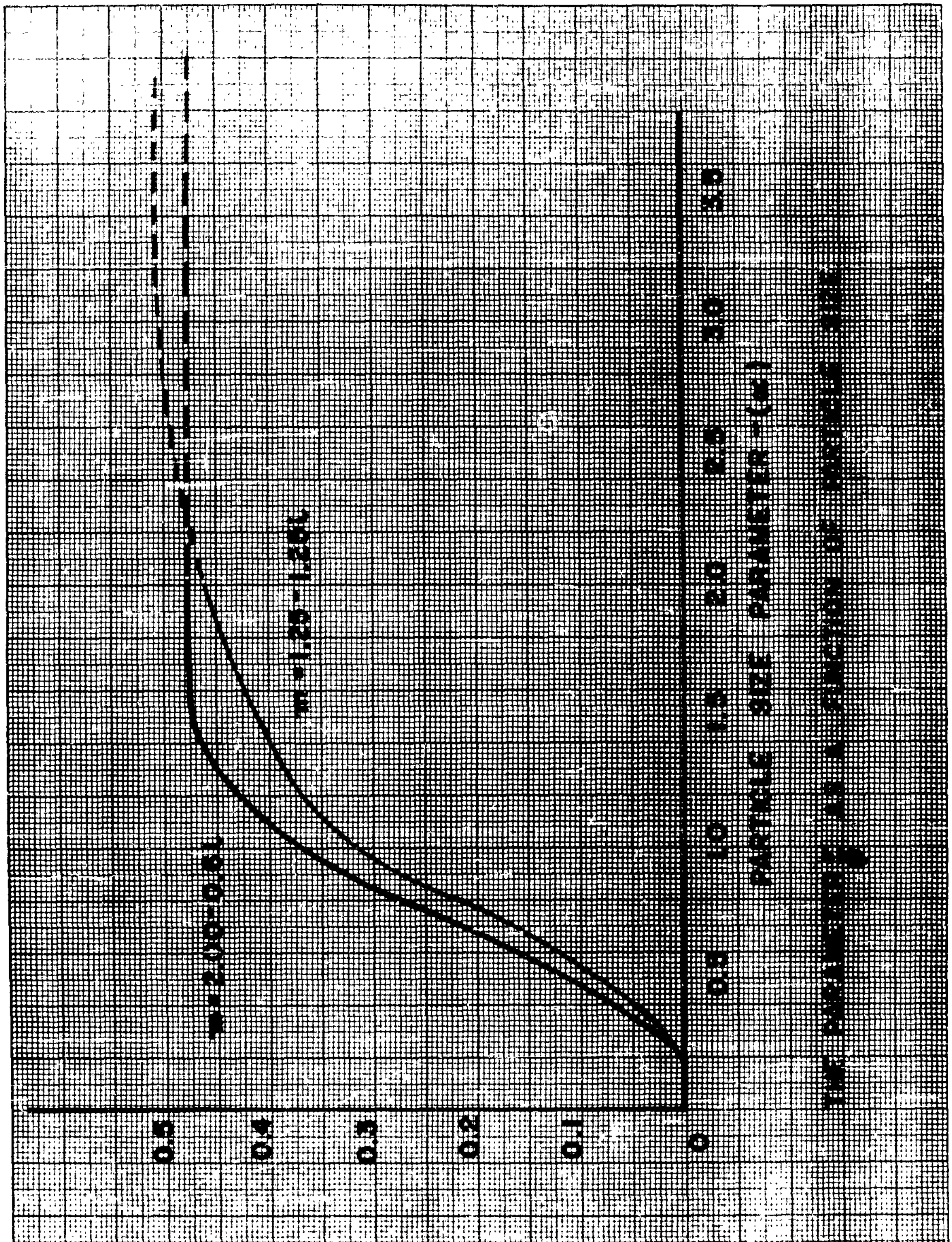

0.5
0.4
0.3
0.2
0.1
0
0.5 1.0 1.5 2.0 2.5 3.0 3.5
PARTICLE SIZE PARAMETER - (α)
m = 2.00 - 0.6i
m = 1.25 - 1.25i
AS A FUNCTION OF PARTICLE SIZE

FIGURE 7

out the oscillations in the scattering function. Thus the scattering function used in this analysis is selected on the basis of the real component of the complex refractive index. A similar approximation was made in the recent work of Churchill, Chu, et al. (8).

The selection of the tables of angular distribution functions by Chu, Clark and Churchill is based upon the range of refractive indices and particle size covered for one degree intervals of scattering angle and the form of presentation. These tables list the scattering function for a range of particle size parameters α, from 1 to 30.

Where:

$$\alpha = \frac{\pi D}{\lambda}$$

D = particle diameter

λ = wave length of radiation

and a range of real refractive indices from 0.9 to 2.0 and .

The table is arranged such that:

$$S(\theta) = 1 + \sum_{n=1}^{n=\infty} a_n P_n (\cos\theta) \tag{93}$$

Where $S(\theta)$ is the axially symmetric scattering function defined previously

a_n is a tabulated coefficient dependent on the particle size parameter α and the refractive index $\bar{m}$

$P_n(\cos\theta)$ is the n^{th} order Legendre Polynomial with the argument $\cos\theta$

The advantage of utilizing this tabulation is that the integration of equation (6)

$$S(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} S(\theta)\, d\varphi' \tag{6}$$

need be carried out only once for all cases.

Thus

$$S(\mu,\mu') = 1 + \frac{1}{2\pi}\sum_{n=1}^{n=\infty} a_n \int_0^{2\pi} P_n(\cos\theta)\, d\varphi' \tag{94}$$

Recalling that:

$$\cos(\theta) = \mu,\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\varphi-\varphi')$$

The integration of equation (94) has been carried out numerically by hand using 10° increments for ω'.

Let

$$P_n(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} P_n(\cos\theta)\, d\omega' \tag{95}$$

Then (94) becomes

$$S(\mu,\mu') = 1 + \sum_{n=1}^{n=\infty} a_n P_n(\mu,\mu') \tag{96}$$

The values of $P_n(\mu,\mu')$ are thus a permanent part of the computer program. The values of a_n must be entered from the tables of Chu, Clark and Churchill for each value of the particle size parameter and refractive index. The limiting value of N replaces the ∞ in the summation since a_n approaches zero as n increases. The number of terms required is approximately 3_α. The integrations for this program were performed up to N = 18.

The table does not list coefficients for particle size α smaller than 1. However, it is well known that particles small compared with wave length scatter according to Rayleigh's law of scattering.

$$S(\theta) = \frac{3}{4}(1 + \cos^2\theta) \tag{97}$$

Therefore for this case

$$a_1 = 0$$

$$a_2 = 0.5$$

For values of a_n corresponding to values of α not given in the table, graphical interpolation is used.

Since a quadrature formula is to be used the scattering function is to be determined for discrete directions. These directions along with the corresponding weight factors as published by Kunz are listed in Appendix F. The values of $P_n(\mu,\mu') = P_n(\mu_i,\mu_j)$ have been computed for each combination of i and j for third and fourth order quadrature approximations, and are tabulated in Appendix G.

Chromey (5) gives values of parameters which are denoted as Ω and Ω' for particle size parameter α from 0.2 to 2.0 in increments of 0.2 and for refractive index $= \bar{m}(1 - ix)$

for $\bar{m} = 0.50, 0.75, 1.00, \ldots, 3.00$
$x = 0, 0.1, 0.2, \ldots, 1.0$

It should be noted that the following relationships exist between Ω and Ω' and the notation of this report.

$$\frac{\sigma}{\beta} = \frac{\Omega'}{\Omega} \tag{99}$$

$$K^e = 2_\alpha \Omega \tag{100}$$

where: K^e = extinction cross section

$$\beta = \frac{3K^e}{4r\rho_a} \tag{101}$$

r = particle radius

ρ_a = mass density of the particle.

For small values of the particle size parameter, an approximation presented by Penndorf (26) may be used:

$$K^e = \frac{24nK_\alpha}{(n^2+K^2)^2 + 4(n^2-K^2) + 4} \tag{102}$$

Where: n and K are defined such that the complex refractive index is $(n - Ki)$. (103)

For values of the particle size parameter larger than $\alpha = 2.0$, the writer found no tables of reasonable completeness in the literature. However, the general behavior may be anticipated from Van De Hulst (33) and other sources and the values are estimated for particle sizes above this value.

It should be noted that polarization of the radiation normally accompanies scattering. However, the effects of polarization, while of great importance in the utilization of scattering for purposes of particle identification, are of small consequence in heat transfer. In particular, since the bounding surfaces are diffuse and their polarizing characteristics generally unknown, little is to be gained from the consideration of polarization during scattering.

5.2 COMPUTATIONS FOR PARALLEL WALLS SEPARATED BY AN ISOTHERMAL MEDIUM

The computations for the radiant heat transfer between parallel walls separated by an isothermal scattering medium were programmed and run on the IBM 650 computer using a fourth order quadrature approximation. The program was written in three parts (1) Computation of the matrix coefficients equation (23), (2) Computation of Eigenvalues and Eigenvectors of the matrix, (3) Computation of the values M, N, and Q as presented in equation (36). The language used was SOAP II for the first and second program, while a machine language program (file number 5.2.018 IBM 650 program library) was used for the Eigenvalues and Eigenvectors.

A check on these computations was then made using the third order quadrature approximation. These computations were made on an IBM 1410 computer utilizing Fortran language. However, the Eigenvectors and Eigenvalues were computed utilizing the same

program as before with the IBM 1410 computer being operated in an IBM 650 simulation mode.

The computed values of M, N, and Q in equation (36) are presented in graphical and tabular form in Appendix B for the fourth order approximation. These values result from numerical computation which proceeds as outlined in Chapter 4. Since heat transfer at the surfaces is normally the region of interest, the program is written to give the values of M, N, Q at $\tau = 0$. It can be seen that M, N, and Q are dependent on the following variables:

(a) ρ_1, the reflectivity of wall (1), the wall at which the radiant heat flux is computed

(b) ρ_2, the reflectivity of wall (2)

(c) τ_o, the optical spacing of the walls

(d) α, the particle size parameter

(e) $\bar{m}$, the complex refractive index

The results of computations are presented for all combinations of the wall reflectivities $\rho_i = 0.1, 0.5, 0.9$, for optical spacings from $\tau_o = 0$ to , for $\alpha = 0$ to 6 and for $\bar{m} = 2.0 - 0.6i$ and $1.25 - 1.25i$. The additional case of isotropic scattering is also presented.

The refractive index 2.0 - 0.6i corresponds to the measured refractive index of carbon at a wave length of 0.546 microns, according to McCartney and Ergun (25). The value of 1.25 - 1.25i corresponds roughly to the refractive index of iron in the visible range. The value of the refractive index is known to vary with wave length. For most materials of engineering interest the value of this property has been of little interest, and published data is not readily available.

In order to compute the integrated radiant flux, reference is made to equation (43). The values of M, N, and Q must be selected from the graphs corresponding to the values of the various independent variables dictated by the ordinates x_j, determined by the quadrature formula. The values of x_j and the weight function a_j are given in Appendix F. Choosing the formula with five ordinates the frequencies and corresponding values of α at which M, N, and Q are determined may be found as follows:

$$\nu = \frac{kx_j}{h} T \tag{104}$$

$$\nu_1 = (0.305 \times 10^{10}) \text{ T sec}^{-1}$$

$$\nu_2 = (1.637 \times 10^{10}) \text{ T sec}^{-1}$$

$$\nu_3 = (4.164 \times 10^{10}) \text{ T sec}^{-1}$$

$$\nu_4 = (8.206 \times 10^{10}) \text{ T sec}^{-1}$$

$$\nu_5 = (14.638 \times 10^{10})\ T\ \mathrm{sec}^{-1}$$

where T is in degrees Rankine.

$$\alpha_j = \frac{\pi k x_j}{ch}\ DT \tag{105}$$

$$\alpha_1 = (\ 0.320 \times 10^{-4})\ DT$$

$$\alpha_2 = (\ 1.714 \times 10^{-4})\ DT$$

$$\alpha_3 = (\ 4.362 \times 10^{-4})\ DT$$

$$\alpha_4 = (\ 8.595 \times 10^{-4})\ DT$$

$$\alpha_5 = (15.334 \times 10^{-4})\ DT$$

where D is particle diameter in microns and T is in degrees Rankine.

It should be cautioned that the temperature used in computing ν and α must be the temperature of wall 1 for the determination of M, wall 2 for the determination of N and of the atmosphere for the determination of Q. Note that the wall reflectivities corresponding to the frequency specified should be used in each case. Thus the walls need not be gray, however, the accuracy is impared if the reflectivity does not vary slowly with wave length.

Based upon the above tabulation of α the extinction coefficient must be determined for each value of α from data such as presented by Chromey (5) and equation (101). The optical spacing of the walls τ_o is then determined corresponding to each α.

With the values of M_j, N_j, Q_j thus determined, the integrated flux q may be expressed as follows:

$$q_{net} = (10^{-11}) \left[T_1^{\,4} \sum_{j=1}^{5} A_j M_j - T_2^{\,4} \sum_{j=1}^{5} A_j N_j - T_a^{\,4} \sum_{j=1}^{5} A_j Q_j \right] \tag{106}$$

where:

$$A_j = \frac{2k^4 a_j x_j^{\,3}}{h^3 c^2 (1-e^{-x_j})} \tag{107}$$

$A_1 = 0.347$ $\quad$ $\mathrm{Btu\text{-}hr^{-1}\text{-}ft^{-2}\text{-}R^{-4}}$
$A_2 = 12.46$
$A_3 = 30.42$
$A_4 = 10.78$
$A_5 = 0.396$ $\quad$ Temperatures must again be in degrees Rankine.

5.3 COMPUTATIONS FOR RADIANT TRANSFER FROM A DIFFUSE WALL TO A SEMI INFINITE MEDIUM

For this case the computed values of M and Q (equation 64 and 65) are represented on the graphs and tables of Appendix B as the limiting value $\tau_o = 5$.

The integration over all frequencies proceeds as above. There being no values of N, the corresponding terms are omitted. Again M_j and Q_j must be evaluated corresponding to the temperature of the wall and atmosphere respectively.

5.4 COMPUTATIONS FOR PARALLEL WALLS SEPARATED BY A MEDIUM IN RADIATIVE EQUILIBRIUM

The values of M, equation (74), for the problem of radiative equilibrium are tabulated in Appendix D. As a result of numerical round off errors the program converged consistently only for the isotropic case. It has been noted that these values are independent of $\frac{\sigma}{\beta}$.

Again the values of M_j and N_j must be determined corresponding to the temperature of surface 1 and surface 2 respectively, and the integration over wave length proceeds as in the previous case.

It should be noted that only two cases of anisotropic scattering converged, (one of these for small $\frac{\sigma}{\beta}$ ratios) $\alpha = 0.3$, $\bar{m} = 1.25 - 1.25i$ and $\alpha = 4.0$, $\bar{m} = 1.25 - 1.25i$. The computed values of M and N for the first case corresponded to those for isotropic case as would be expected. For the second case, the values were inconsistent indicating problems with round off errors in the matrix inversion. However, averaged values for the parameters fall relatively close to the values for the isotropic case.

5.5 COMPUTATIONS FOR THE PROBLEM OF A NORMAL INCIDENT FLUX

The problem of a normal incident flux described in 4.4 was computed utilizing an IBM 1410 computer with the Eigenvalues and Eigenvectors from the previous program being used. Fortran language was utilized, and it was necessary to write the program in two parts because of computer storage limitations.

Computations were made for the same range of optical thicknesses, particle size parameters, refractive indices and surface reflectivities as in the previous problems. The results of the computation are presented in Appendix E. These correspond to the monochromatic flux at the opaque surface.

5.6 DISCUSSION OF NUMERICAL RESULTS

5.6.1 Accuracy of Results

An exact statement concerning the accuracy of the results of this study can not be made since the results of an exact analysis are not available for comparison. However, comparison of the computations based on a third order quadrature approximation

with those of the fourth order and the consistency of the results over a number of different computations indicate that the method results in an accuracy suitable for engineering computations.

For the isotropic case of conservative scattering in a semi-infinite atmosphere, Sykes (32) has demonstrated that the maximum error for the intensity, computed using the third order quadrature, is 0.7%. The error for the average intensity being much less. Additional errors might be expected in the approximation of the scattering function, however, for the anisotropic case.

Accuracy of the results may be influenced by two principal types of errors:

(a) Errors in the approximation of the function by quadrature
(b) Errors resulting from numerical round off procedures in the computation.

In order to obtain an indication of the effect of the order of quadrature approximation, computations were made for certain cases utilizing a third order approximation for purposes of comparison with the fourth order approximation. The results of the third order computation are tabulated in Appendix C.

A comparison of these results with those in Appendix B indicates a good agreement, the computed values differing less than a significant figure in the hundreths place in most instances.

The effect of numerical round off errors are not believed to be significant because of the consistency of the computations over the various cases computed. The two exceptions being the problems encountered in the case of radiative equilibrium as mentioned above and in a single case in the problem of the normal incident radiation. The later case occurred for the case of $\alpha = 4$, $m = 1.25 - 1.25i$. An intermediate read out of ξ_i indicated the values to be much larger (a factor of 100) than other cases. As mentioned in the discussion of the numerical procedure this may be expected for some values, and may result in round off errors in the subsequent computations.

5.6.2 Comparison of Results

Figure 8 and 9 show comparative plots of the parameter Q and Q^n for the cases of radiant heat transfer between parallel places and normal incident flux respectively. The graphs are typical of the result of both types of problems. In both cases it can be seen that under certain conditions, the radiant heat transfer through the media consisting of particles of refractive index 2.00 - 0.6i and 1.25 - 1.25i respectively will be approximately the same as through media consisting of particles which have the same $\frac{\sigma}{\beta}$ ratio and scatter isotropically. These conditions are (1) when the optical thickness of the cloud is small and (2) when the particle size parameter, α is less than 1.0.

This indicates a very significant possibility that for optically thin systems the computation of radiant heat transfer may be performed treating the scattering as though it were isotropic.

At large optical thicknesses, however, the deviation from the isotropic case

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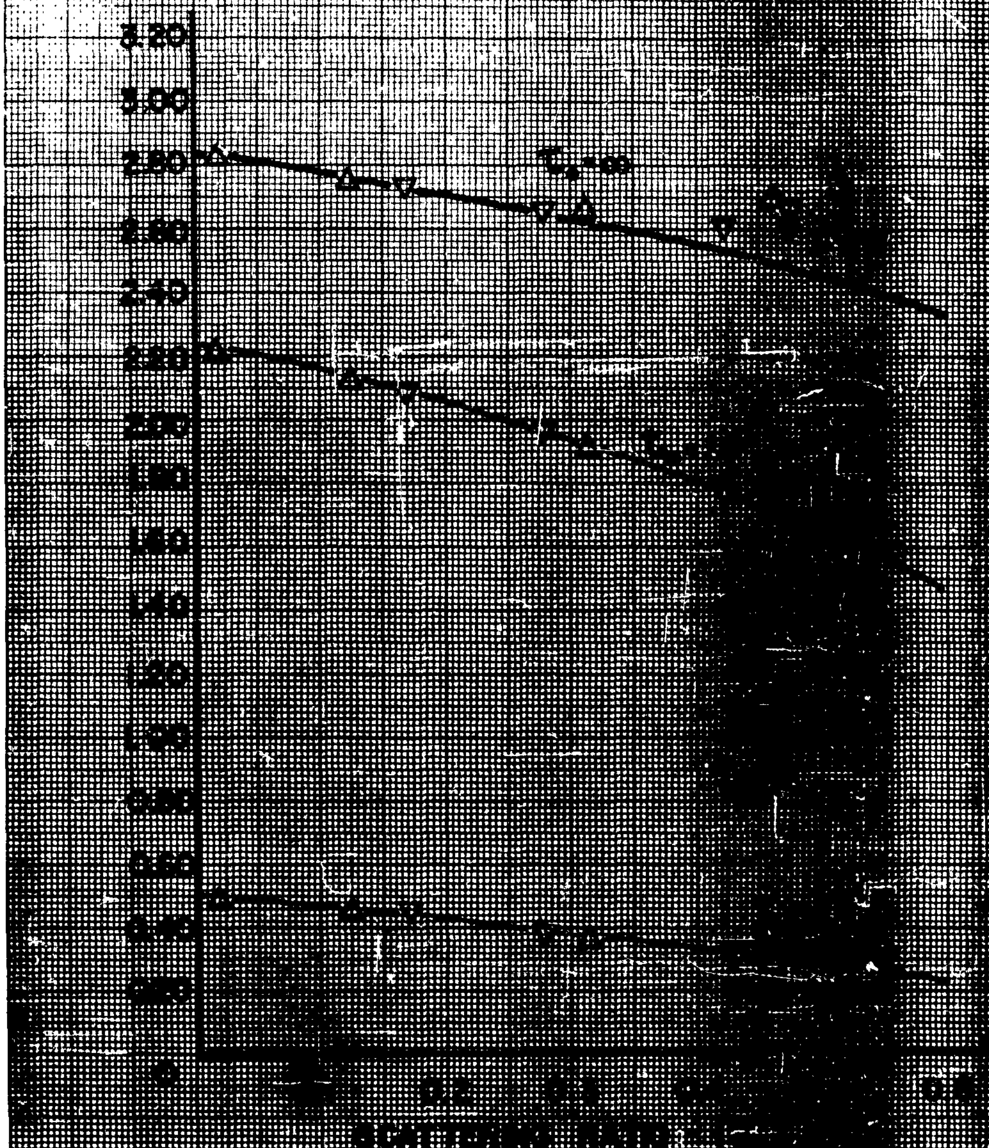
5.6.2 Comparison of Results

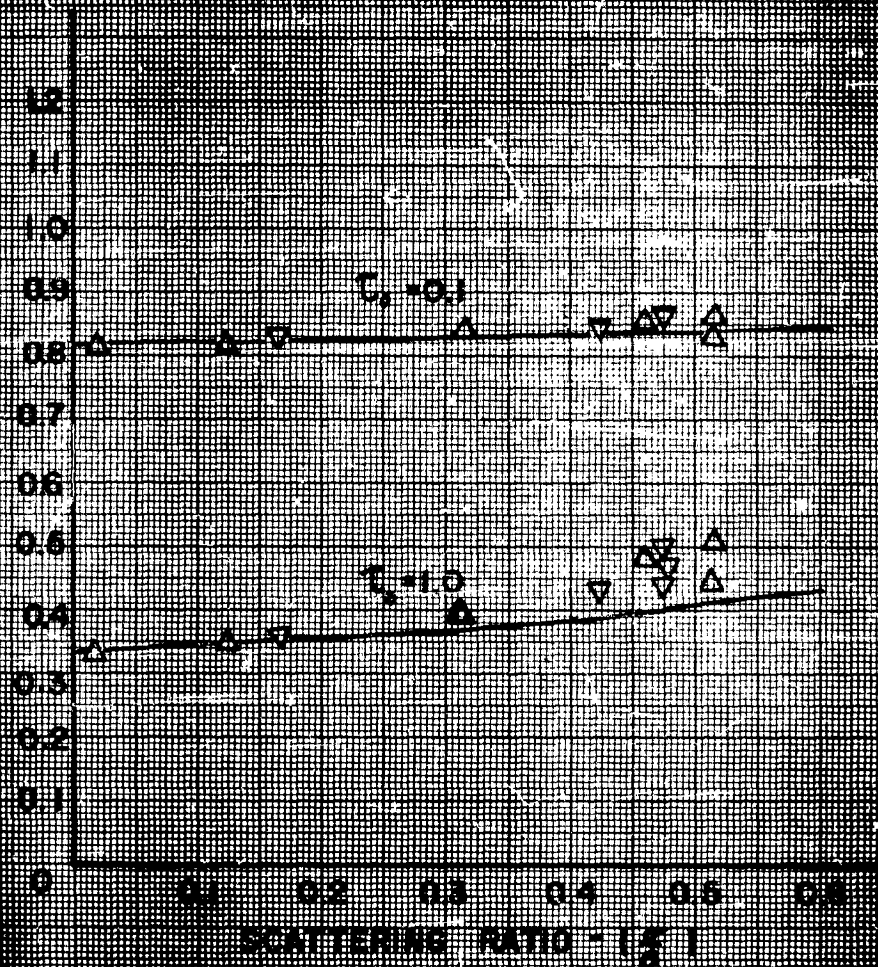
Figure 8 and 9 show comparative plots of the parameter Q and Q^n for the cases of radiant heat transfer between parallel places and normal incident flux respectively. The graphs are typical of the result of both types of problems. In both cases it can be seen that under certain conditions, the radiant heat transfer through the media consisting of particles of refractive index 2.00 - 0.61 and 1.25 - 1.25i respectively will be approximately the same as through media consisting of particles which have the same $\frac{\sigma}{\beta}$ ratio and scatter isotropically. These conditions are (1) when the optical thickness of the cloud is small and (2) when the particle size parameter, α is less than 1.0.

This indicates a very significant possibility that for optically thin systems the computation of radiant heat transfer may be performed treating the scattering as though it were isotropic.

At large optical thicknesses, however, the deviation from the isotropic case



COMPARISON OF PARAMETER [illegible] FOR [illegible] VS
ANISOTROPIC SCATTERING
(CASE OF PARALLEL WALLS)
$\rho_1 = 0.1$ $\rho_2 = 0.1$ ISOTROPIC —— [illegible] -△
2.00-0.60L-▽
FIGURE 8



COMPARISON OF PARAMETER Q^* FOR ISOTROPIC VS. ANISOTROPIC SCATTERING (CASE FOR NORMAL INCIDENT FLUX).

[illegible]

ISOTROPIC ———

[illegible] △

[illegible] ▽

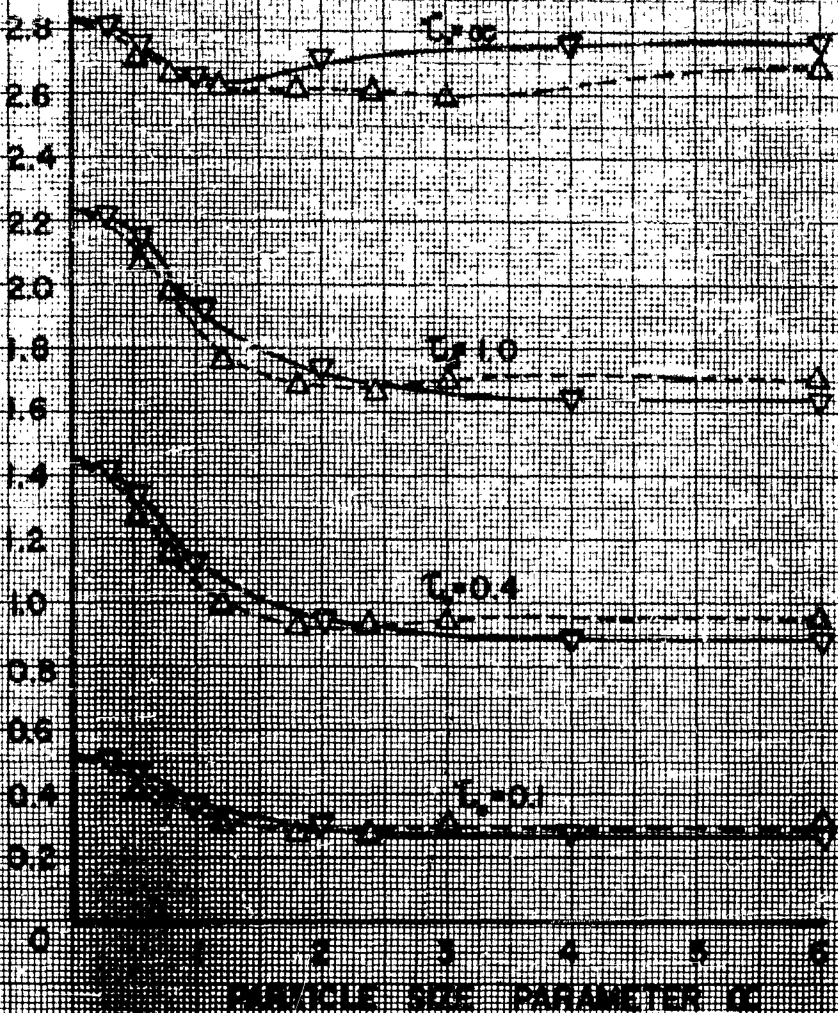
FIGURE 9

is significant. It should be noted that the computed results for the two refractive indices examined tend to be relatively close for the same value of the particle size parameter as shown in Figure 10.

5.6.3 Extrapolation of Results for Large Values of α

While the computations were carried out only up to values of the particle size parameter of 6, the results indicate little variation of the computed quantities for values of α greater than 3. This is not surprising since the ratio $\frac{\sigma}{\beta}$ is approximately constant for larger values of α.

The change of the scattering function with increasing particle size is one of retaining the same general shape while the scattered intensity fluctuates with an increasing frequency as α increases. In the radiant heat transfer problem, however, the diffuse nature of the radiation and the "average" nature of the flux determination tends to decrease the effects of these "local" fluctuations in the scattering function.



COMPARISON OF EFFECTS OF REFRACTIVE INDEX ON PARAMETER Q.

1.25 - 1.25 i

2.00 - 0.60 i

FIGURE 10

6. DISCUSSION OF POSSIBLE EXPERIMENTAL DETERMINATION OF SCATTERING FUNCTIONS

It has been noted that the analytical determination of scattering functions is feasible only for a relatively small class of geometrical particle shapes. In addition, for dispersions containing a range of particle sizes, the scattering function must be computed by a super position technique which sums the individual scattering from the concentration of each particle size present.

Because of the complexity of the particle shapes, size distribution and the variation of refractive index with wave length in dispersions which are likely to be encountered in systems of engineering interest an experimental study of scattering functions is desirable. In actual application of the analysis of this study, it is expected that an experimental determination of the scattering function may be required for the particular medium of the application.

The scattering from such a media is expected to be a relatively smooth function of angle on the basis of reports given recently at the Interdisciplinary Conference on Electromagnetic Scattering held at Clarkson College of Technology in August, 1962, jointly sponsored by the American Chemical Society and the Air Force Cambridge Research Laboratories. (The proceedings of the conference are expected to be published within the next year.) Investigations show that the scattering by such systems is a relatively smooth function of angle. In addition, analytical studies of the scattering by a system of spherical particles composed of a range of sizes indicate that again the scattering will be a relatively smooth function of angle.

The experimental determination of the scattering function is visualized as being accomplished with the utilization of a relatively "white" infra-red source and a standard monochromator. It is expected that numerous difficulties will have to be overcome in the development of the experimental techniques. Some of the anticipated problems are discussed below.

Preparation and retention of a sample of the scattering medium represents a significant problem. In order to obtain the single scattering effects, the sample must be optically thin which in turn results in a relatively weak scattered intensity. The sample must not be confined within a restrictive container such as a glass tube because of the problems of surface reflections. It appears that the sample must, therefore, either be a continuous stream of falling particles or a stream of particles conveyed by a vertical stream of air and collected by a suction device above the sample point. In either case attention must be paid to assure a uniformly dense and representative sample throughout the test.

The sample would then be irradiated by infra-red radiation from a source such as a "glow bar." Through suitable optics mounted on a table which would rotate about the vertical axis of the sample column. The source radiation would be focused through an off-axis parabolic mirror and collimated. The incident beam should be "chopped" to provide a means of filtration and amplification of the signal.

A monochromator would be positioned with suitable optics for focusing the scattered radiation on the monochromator slit.

The ratio of the scattered radiation to the incident intensity as a function of wave length would then be measured at discrete angles which would be determined by experiment.

The extinction coefficient should be measured directly by determining the decrease of intensity of the radiation as it traverses a known mass depth of the sample.

An emission coefficient for the particles could be obtained by heating the particles and the entraining air stream to a relatively high temperature and measuring the emitted intensity of the stream of particles.

It must be recognized that such an experimental procedure as outlined above would by no means be a straight forward procedure but will require considerable development.

7. SUMMARY AND CONCLUSIONS

The problem of radiant heat transfer through plane parallel clouds of scattering particles has been solved, and numerical results have been obtained for the following cases:

(a) Radiant heat transfer between reflecting walls separated by a scattering, emitting, and absorbing isothermal medium.

(b) Radiant heat transfer from a reflecting wall to a semi-infinite scattering, emitting, and absorbing isothermal medium.

(c) Radiant heat transfer between reflecting walls separated by a scattering, emitting, and absorbing medium in radiative equilibrium.

(d) Radiant heat transfer at a reflecting wall bounded by a scattering, emitting, and absorbing medium of finite thickness with a normal incident flux.

Results are tabulated for the cases of clouds of uniform spherical particles with refractive indices of 1.25 - 1.25i (corresponding roughly to metallic particles), and 2.00 - 0.6i (corresponding roughly to carbon particles) over a range of particle size to wave length ratio, and the case of isotropic scattering over a range of scattering to extinction ratios. The computations include combinations of wall reflectivities and the method of integration over wave length permits non-gray surfaces.

The conclusions which may be drawn from this study may be listed as follows:

(a) The approximate numerical method developed in this work gives consistent results which appear to be within normal accuracy expected of engineering heat transfer calculations.

(b) The method is restricted to plane parallel axially symmetric geometries, however, qualitatively the effects of scattering may be extended to other systems. For the case of optically thick systems, the problem of the semi-infinite medium will give quantitative results.

(c) For optically thin media and media composed of particles which are smaller than the wave length the approximation of isotropic scattering appears to give good results.

(d) Infinitely thick clouds of uniform particles can not be assumed black, particularly in the wave length range corresponding to the particle size.

(e) The effect of particle scattering seems to be independent of particle size for particles large compared to the wave length of the radiation.

(f) The effect of scattering on the overall heat transfer is significant even for small optical separations of plates.

This study has been in no sense an exhaustive one and there appears to be endless possibilities for continuing research in this area. In particular the following areas are suggested for further investigation:

(a) Extension of this analysis or development of a method of study for other geometries.

(b) Study of scattering functions of particles of random shapes and sizes.

(c) Extension of analysis to non-isothermal systems.

(d) Studies of combined conduction, convection and radiation in scattering systems, particularly thermal insulations.

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APPENDIX A

EXAMPLE PROBLEM

Statement of Problem

Two infinite plane parallel walls are spaced 0.2 ft. apart. A uniform cloud of iron particles occupies the intervening space. The apparent density of the cloud is 0.01 $(lb_m)(ft)^{-3}$. The particles are assumed to be spheres with a diameter of 2 microns and a refractive index of 1.25 - 1.25i for all frequencies. The cloud moves past the area of interest in a rapid turbulent fashion such that the cloud may be considered isothermal. The following conditions of temperature and wall properties exist.

Wall 1 -- temperature, 2000°R, reflectivity, 0.10

Wall 2 -- temperature, 500° R, reflectivity, 0.90

Particles -- temperature, 1000° R

Information Desired

Find the net radiant flux at Wall 1.

Solution

(a) Compute the 3 sets α_j corresponding to the 2 micron particle diameter and the respective temperatures of Wall 1, Wall 2, and the atmosphere. This is accomplished with the utilization of equation (105).

(b) From graph figure 6, read K_j^e corresponding to each α_j computed above.

(c) Compute $(\tau_o)_j$ corresponding to each K_j^e computed above.

(d) Read M_j, N_j, Q_j from the graphs in Appendix B. The values of M_j should correspond to the values of α_j and $(\tau_o)_j$ computed using the temperature of wall 1. The values of N_j should correspond to the temperature of wall 2, and Q_j should correspond to the temperature of the particles. These values should be read from the graphs corresponding to $\rho_1 = 0.10$ and $\rho_2 = 0.90$ and the refractive index of the particles, $m = 1.25 - 1.25i$.

The results of the above operations for this problem are tabulated below.

Corresponding to wall 1 temperature of 2000° R

j	α_j	K_j^e	$(\tau_o)_j$	M_j
1	0.13	0.38	0.353	2.17

Corresponding to wall 1 temperature of 2000° R (continued)

j	α_j	K_j^e	$(\tau_o)_j$	M_j
2	0.69	2.21	2.051	2.71
3	1.74	2.92	2.710	2.68
4	3.44	2.64	2.45	2.69
5	6.13	2.44	2.26	2.71

Corresponding to wall 2 temperature of 500° R

j	α_j	K_j^e	$(\tau_o)_j$	N_j
1	0.032	0.08	0.074	0.265
2	0.171	0.52	0.483	0.130
3	0.436	1.35	1.253	0.050
4	0.860	2.62	2.431	0.019
5	1.530	2.89	2.682	0.021

Corresponding to particle temperature of 1000° R

j	α_j	K_j^e	$(\tau_o)_j$	Q_j
1	0.064	0.19	0.176	1.17
2	0.340	1.05	0.974	2.60
3	0.872	2.63	2.440	2.65
4	1.72	2.91	2.700	2.64
5	3.07	2.69	2.500	2.65

(e) The values M_j, N_j, Q_j are then multiplied by the corresponding value A_j presented in equation (107). The resulting terms are summed and multiplied by the fourth power of the corresponding temperature.

	A_jM_j	A_jN_j	A_jQ_j
	0.75	0.09	0.41
	33.77	1.62	32.46
	81.68	1.52	80.77
	29.05	0.20	28.46
	1.07	0.01	1.05
Total	146.32	3.44	143.14

$$q_{net} = (10^{-11})\left[(146.32)(16\text{x}10^{12}) - (3.44)(0.0625)(10^{12}) - (143.14)(10^{12})\right]$$

$$q_{net} = 21{,}960 \ (\text{Btu})(\text{hr})^{-1}(\text{ft})^{-2}$$

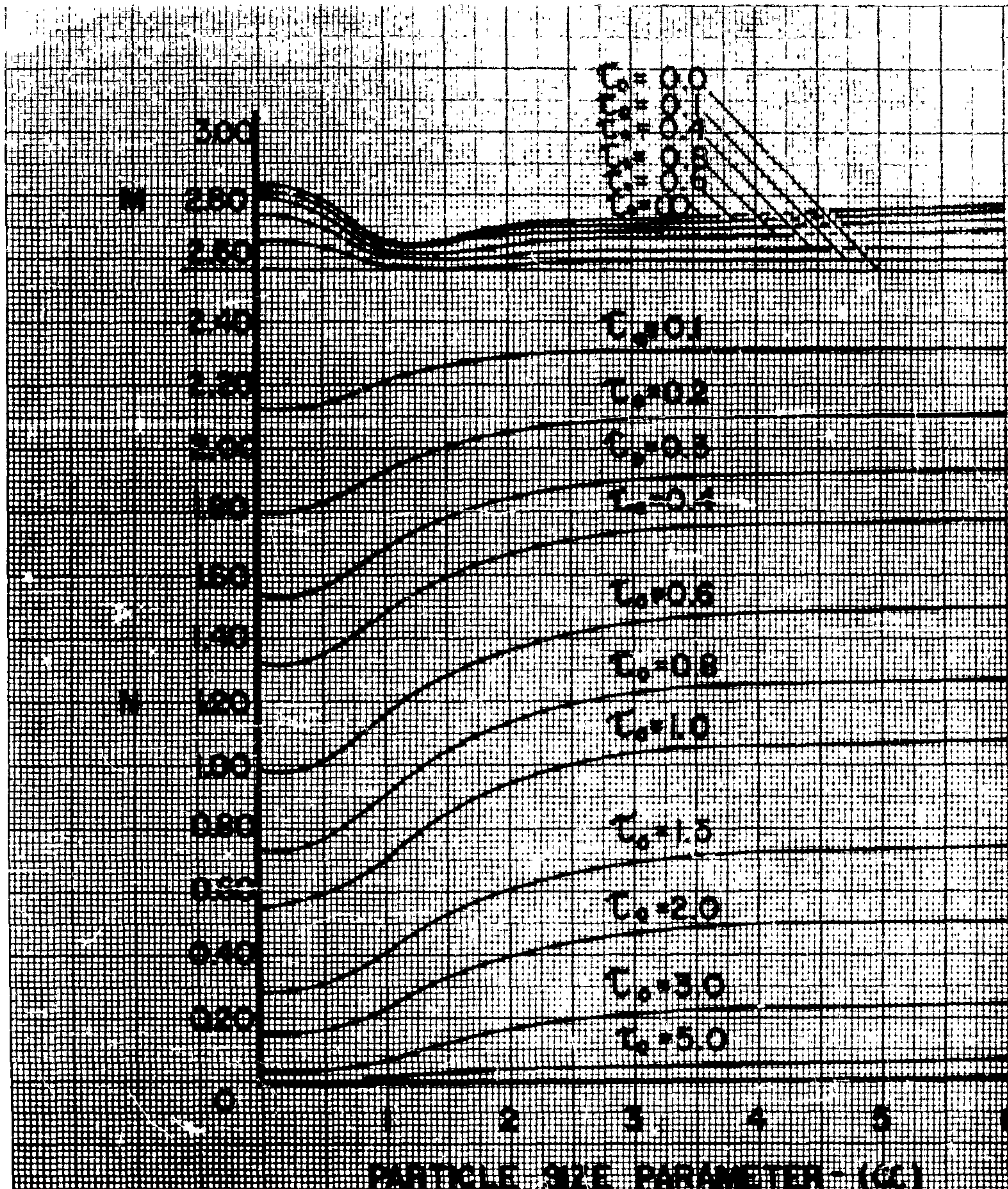
APPENDIX B

Graphical and Tabulated Results

for

Parallel Plates

Fourth Order Approximation

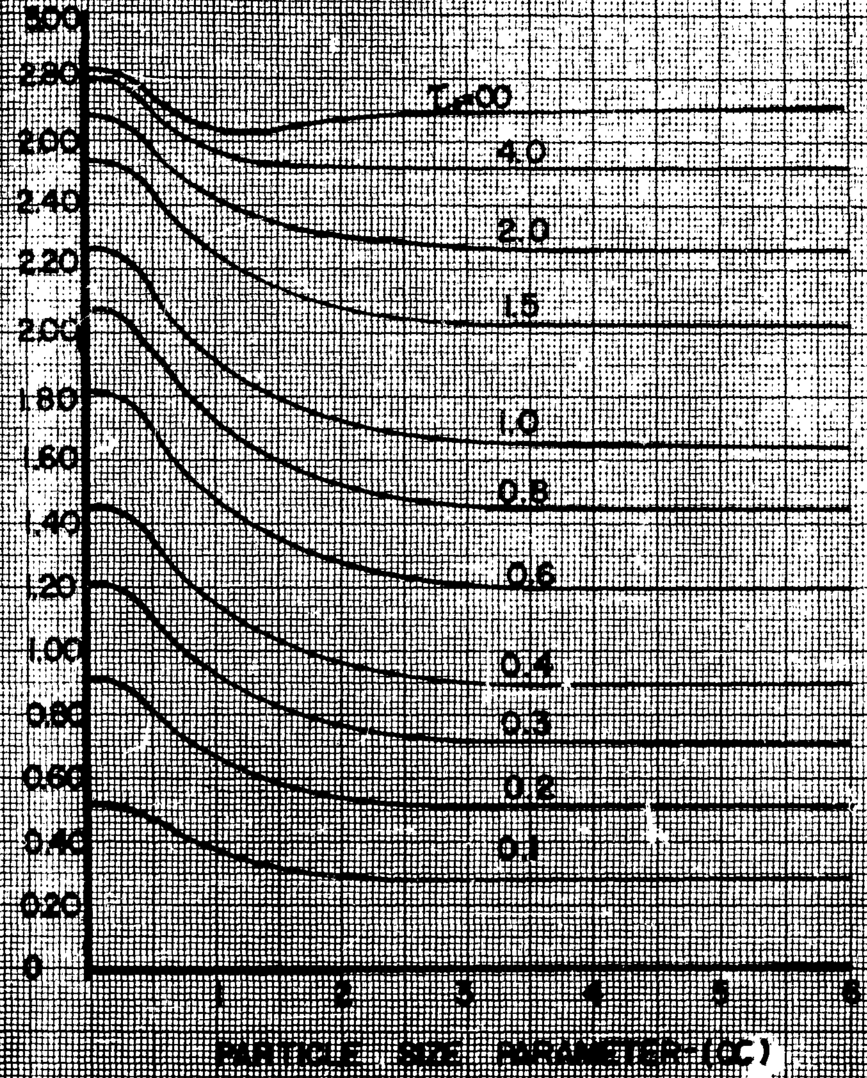


VALUES OF PARAMETER

M & N

$\rho_1 = 0.1$ $\rho_2 = 0.1$ $m = 1.25 - 1.25i$

FIGURE 11

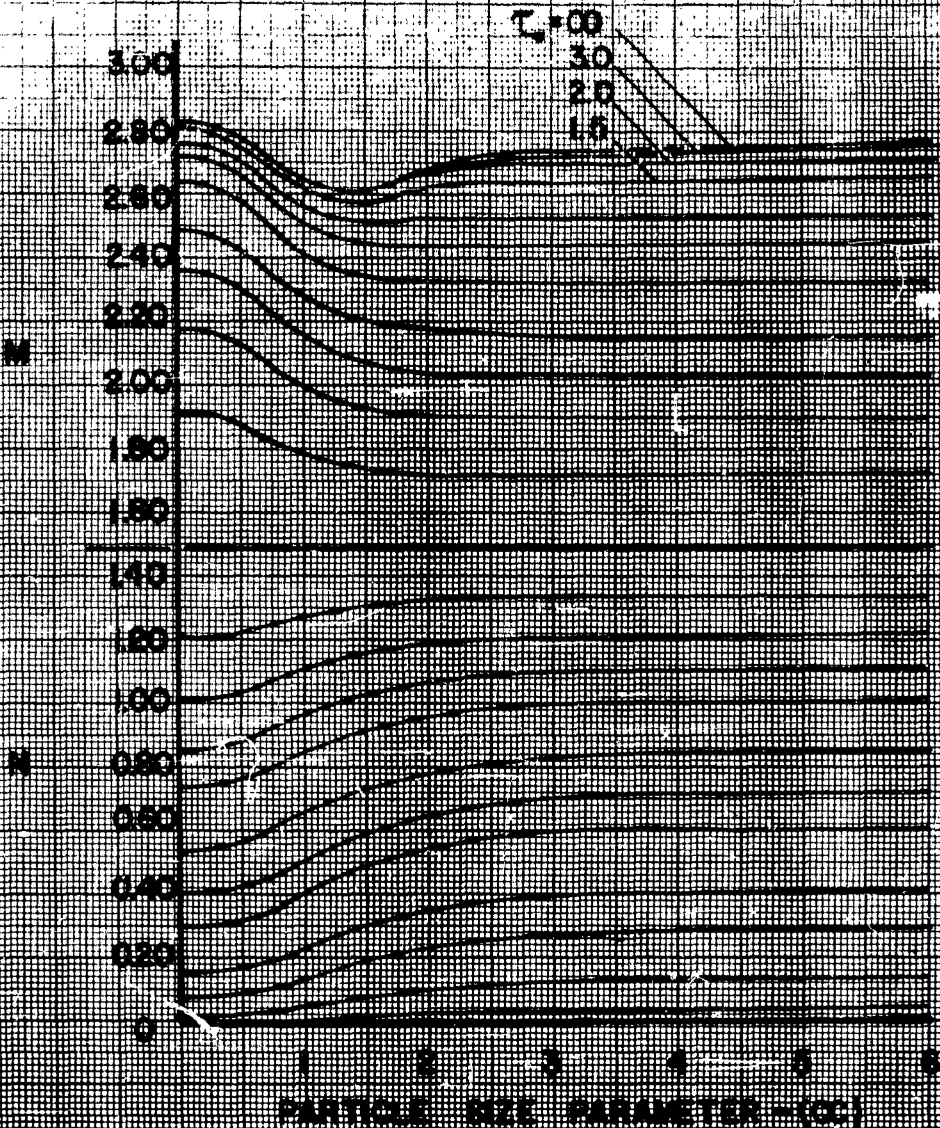


VALUES OF PARAMETER

Q

$\rho_1 = 0.1 \quad \rho_2 = 0.1 \quad m = 1.25 - 1.25i$

FIGURE 12

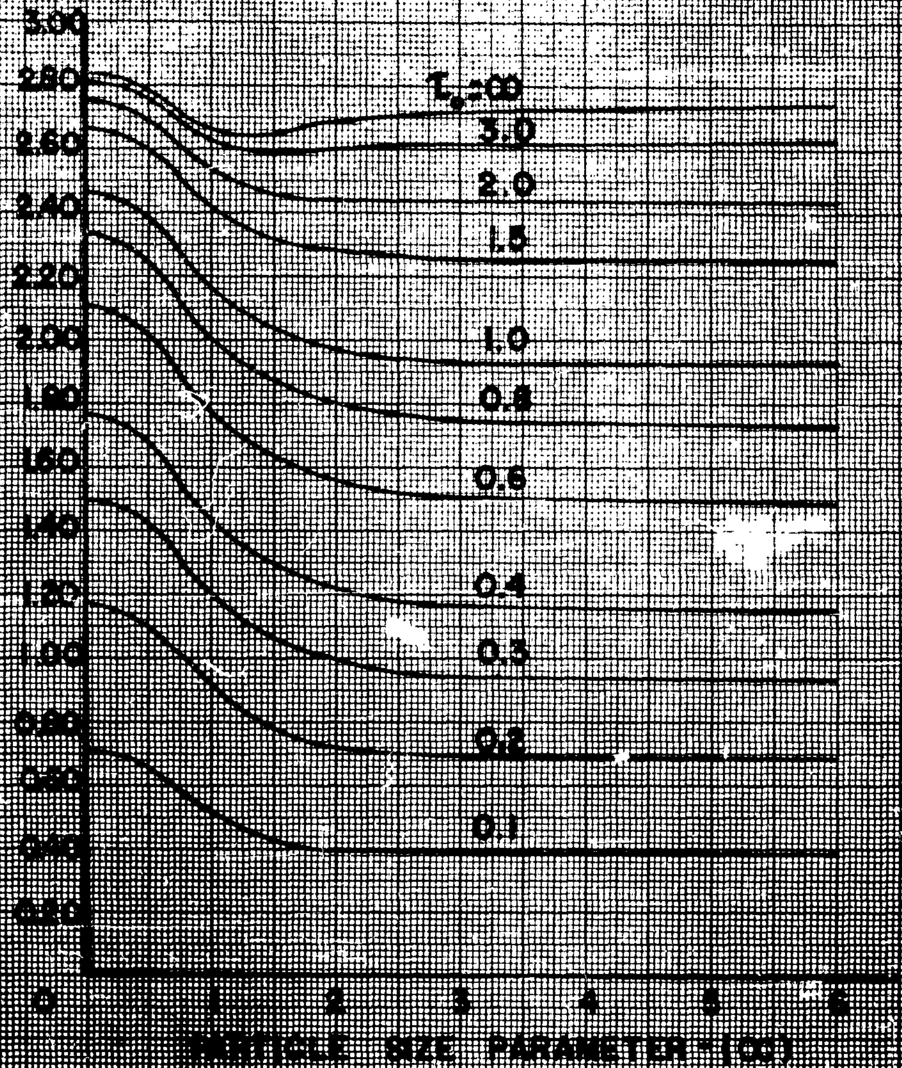


VALUES OF PARAMETERS

N & R

$\rho_1 = 0.1 \quad \rho_2 = 0.25 \quad m = 1.25 - 1.25i$

FIGURE 13

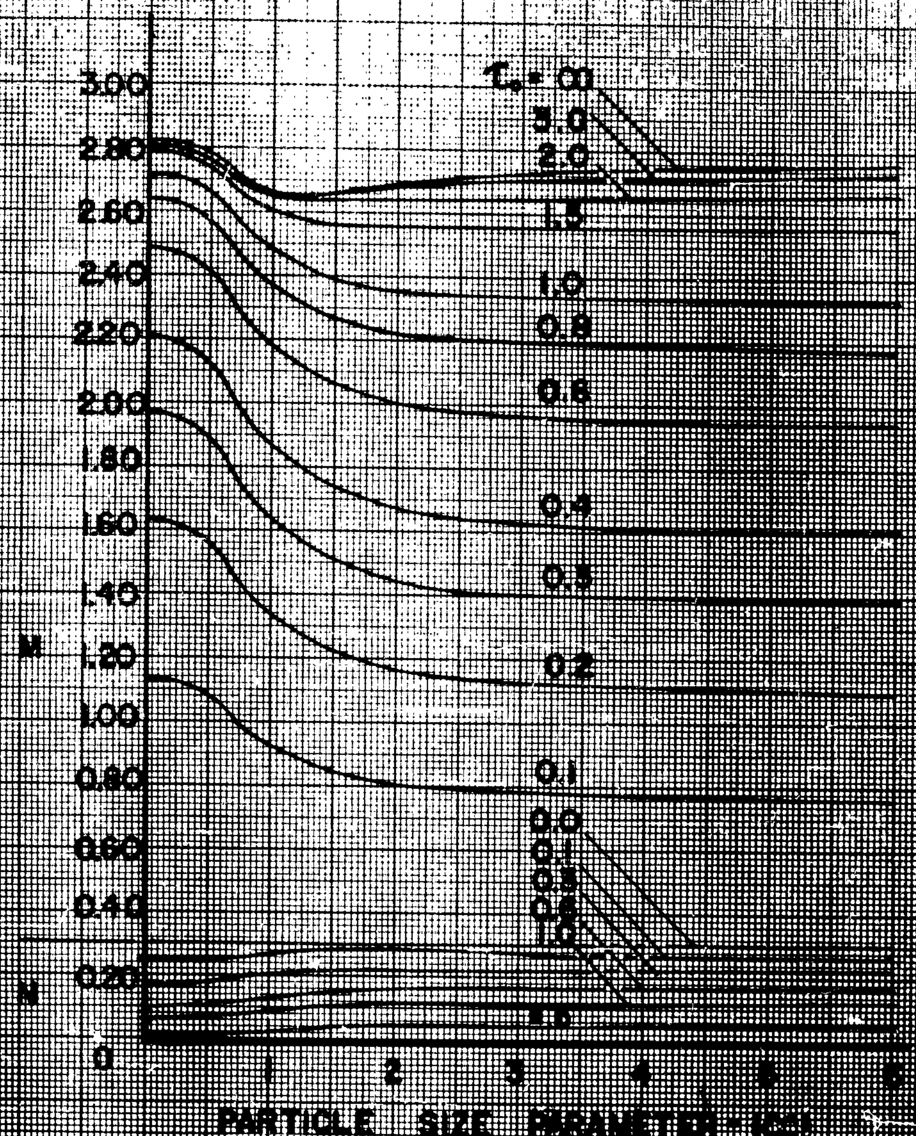


VALUES OF PARAMETER

Q

$\rho_1 = 0.1 \quad \rho_2 = 0.5 \quad m = 1.25 - 1.25i$

FIGURE 14

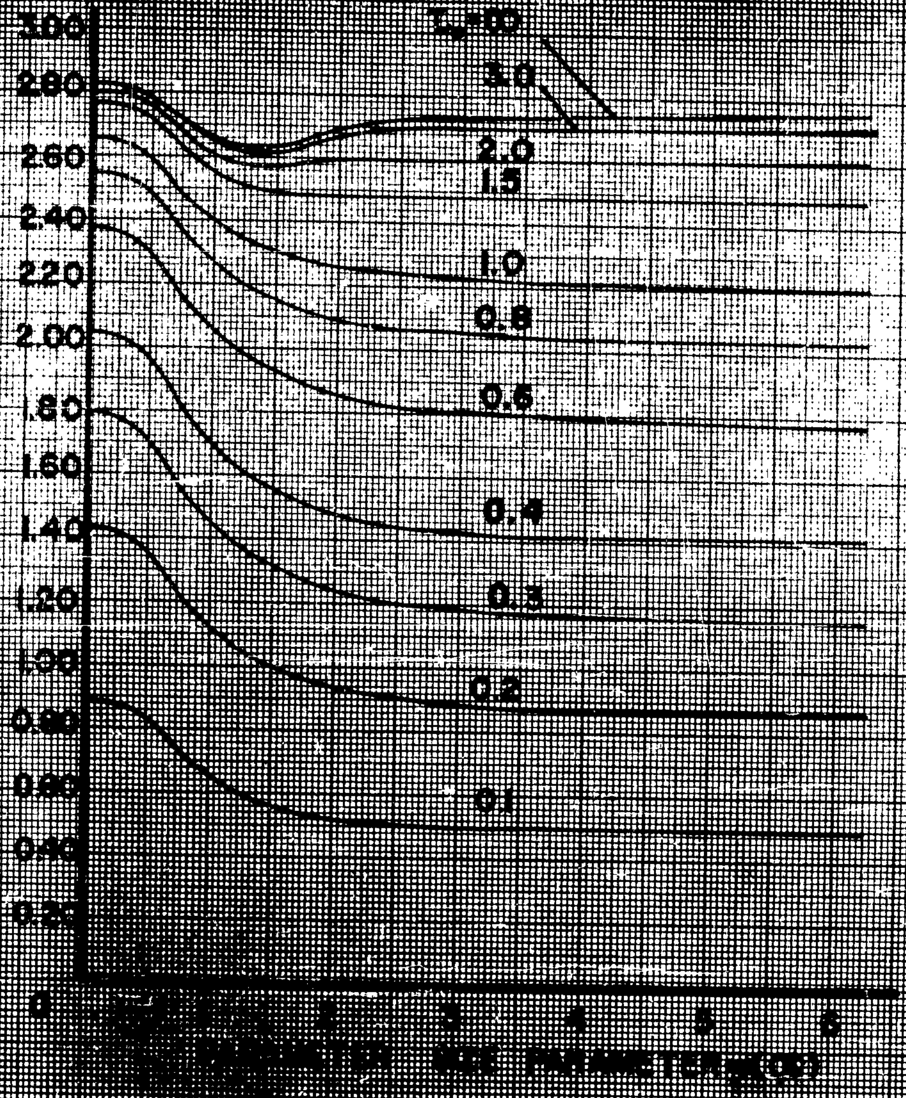


VALUES OF PARAMETERS

M & N

$\rho_1 = 0.1$ $\rho_2 = 0.9$ m = 1.25 - 1.25 i

FIGURE 15

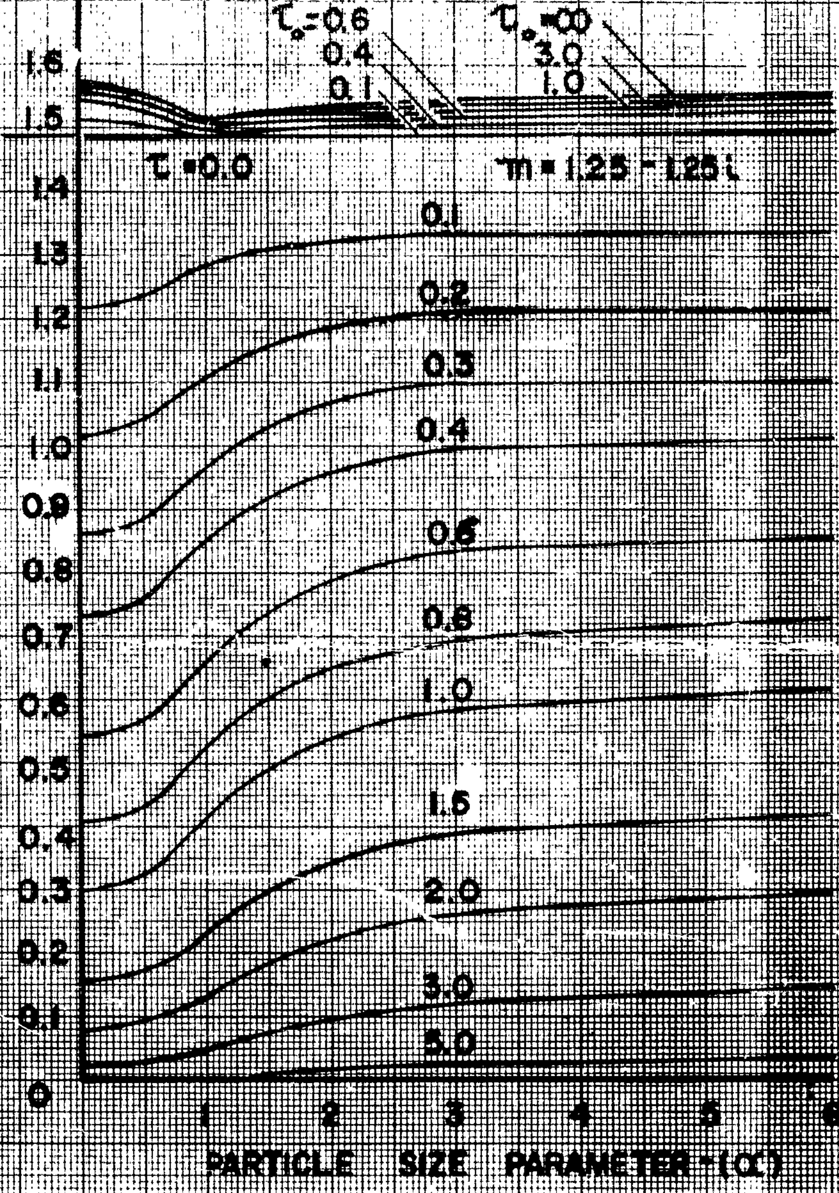


VALUES OF PARAMETER

Q

$\rho_1 = 0.1$ $\rho_2 = 0.9$ $m = 1.25 - 1.25i$

FIGURE 16

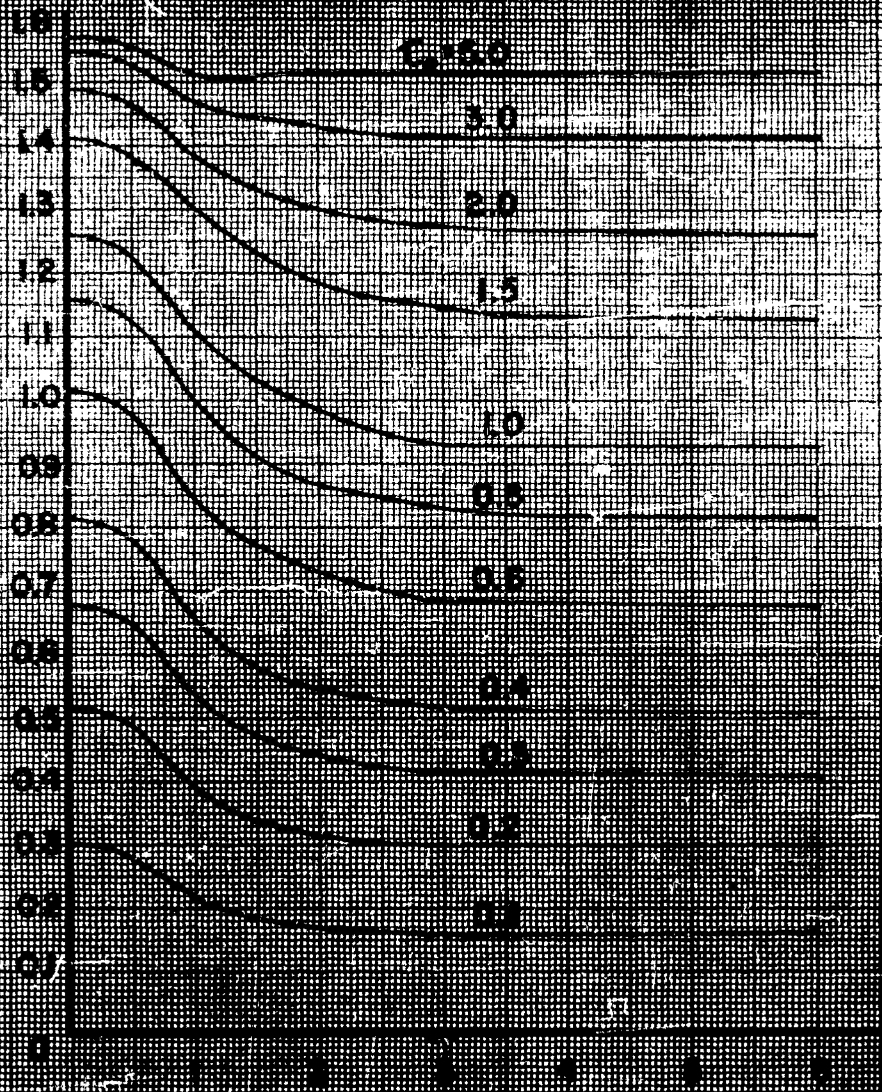


VALUES OF PARAMETERS

M & N

ρ_1 = 0.5 ρ_2 = 0.1

FIGURE 17



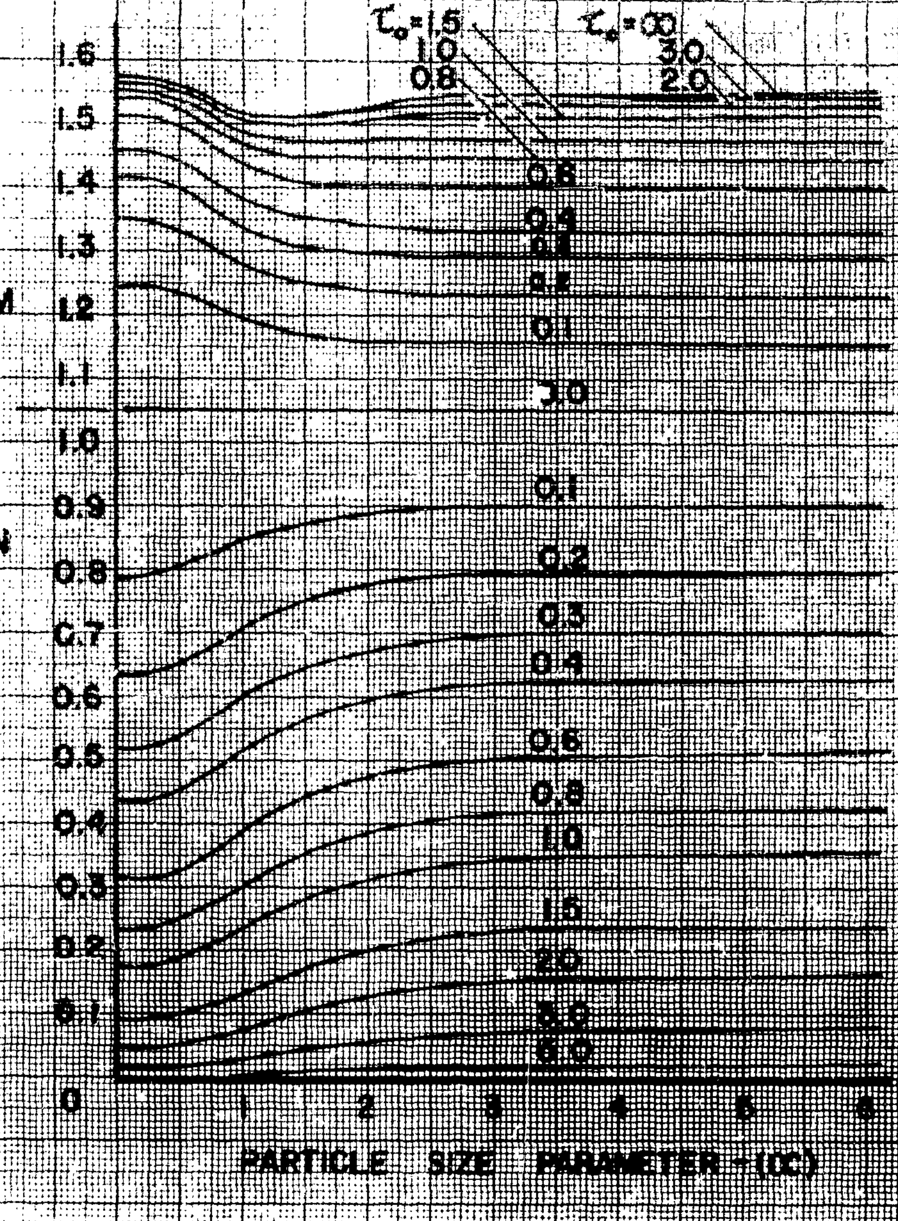
PARTICLE SIZE PARAMETER (α)

VALUES OF PARAMETER

C

$\rho_1 = 0.5 \quad \rho_2 = 0.1 \quad m = 1.25 - 1.25i$

FIGURE 18

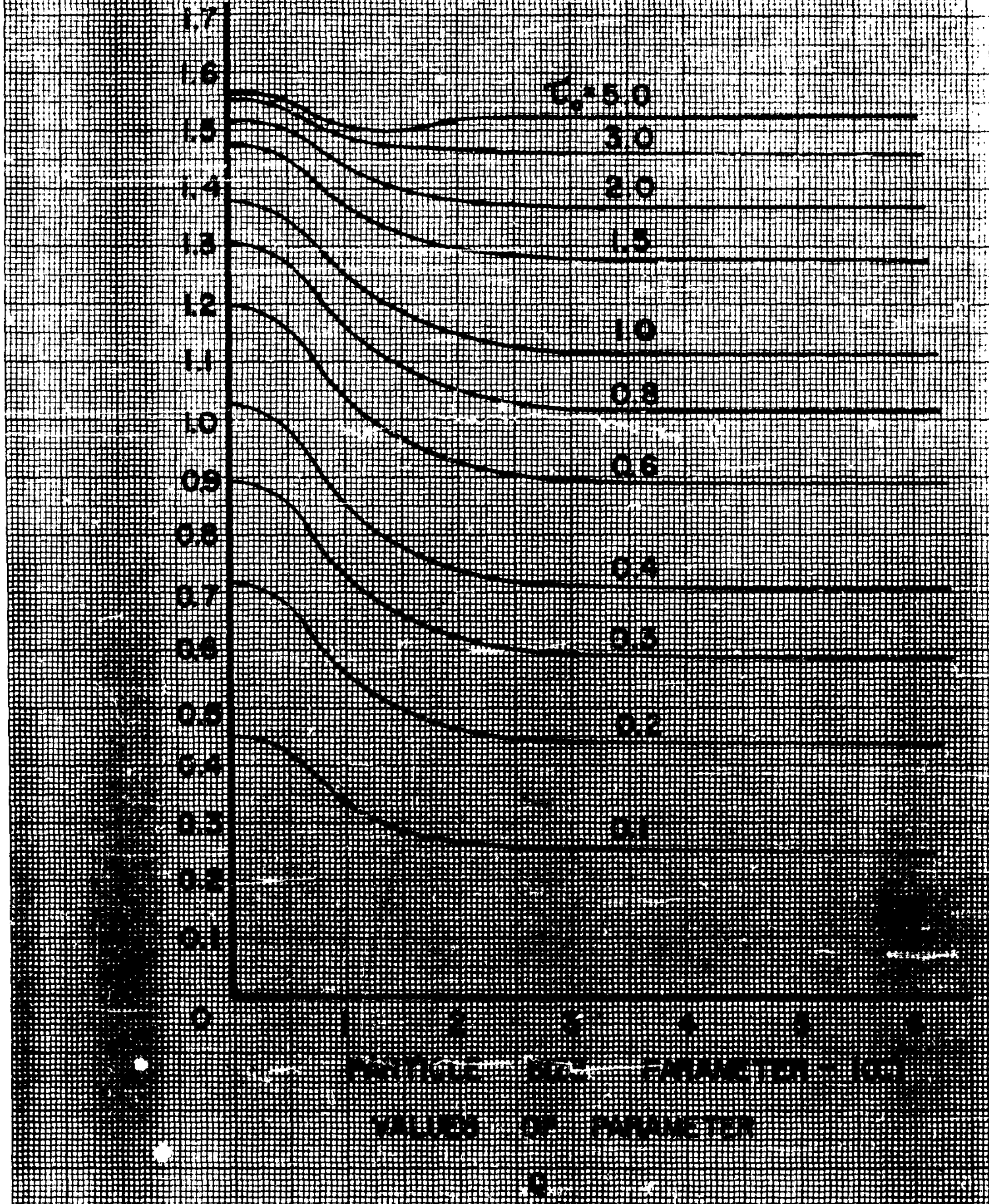


VALUES OF PARAMETERS

M & N

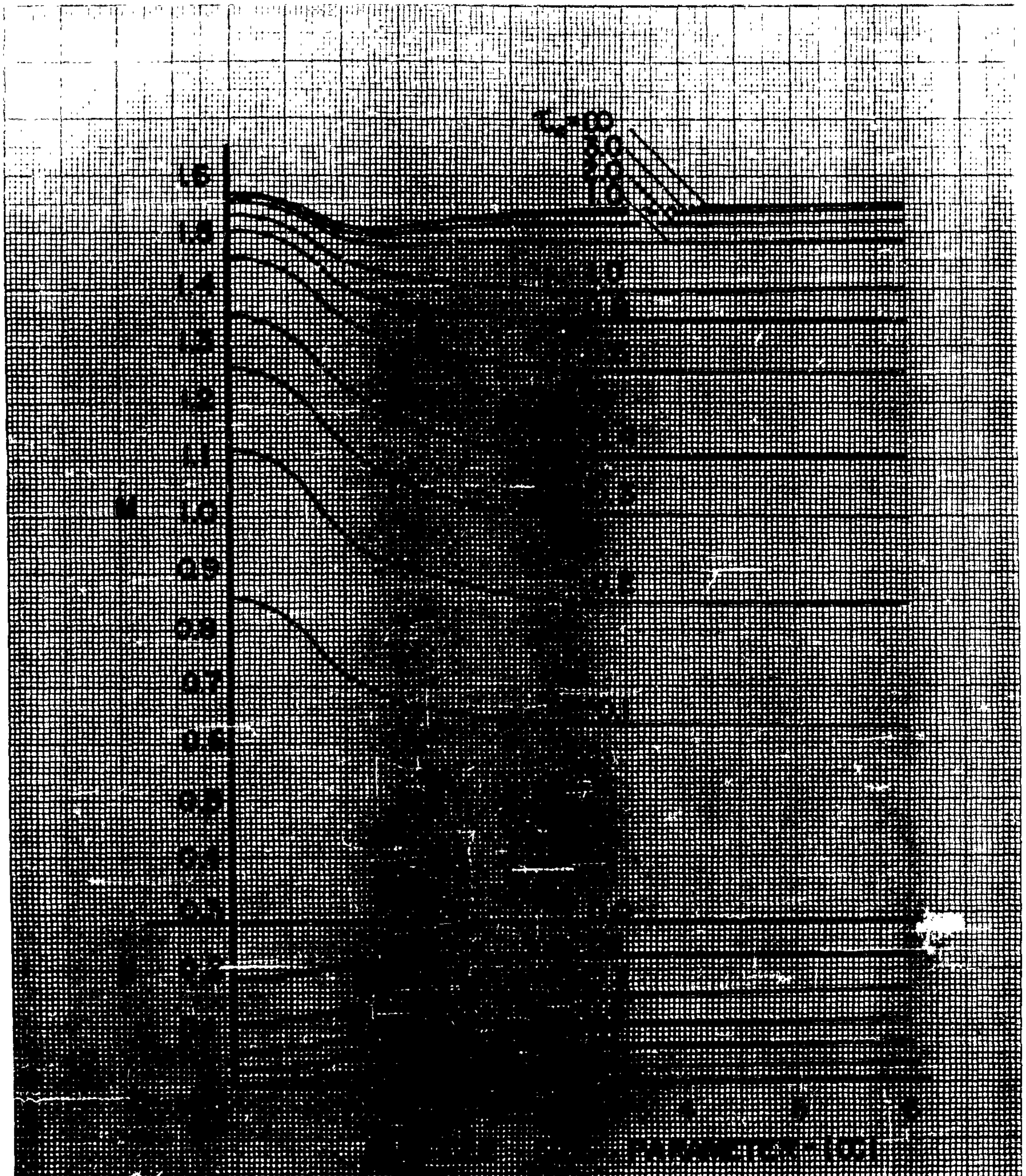
$\rho_1 = 0.5$ $\rho_2 = 0.5$ $m = 1.25 - 1.25i$

FIGURE [illegible]



$\rho_1 = 0.5 \quad \rho_2 = 0.5 \quad m = 1.25 - 1.25i$

FIGURE 20

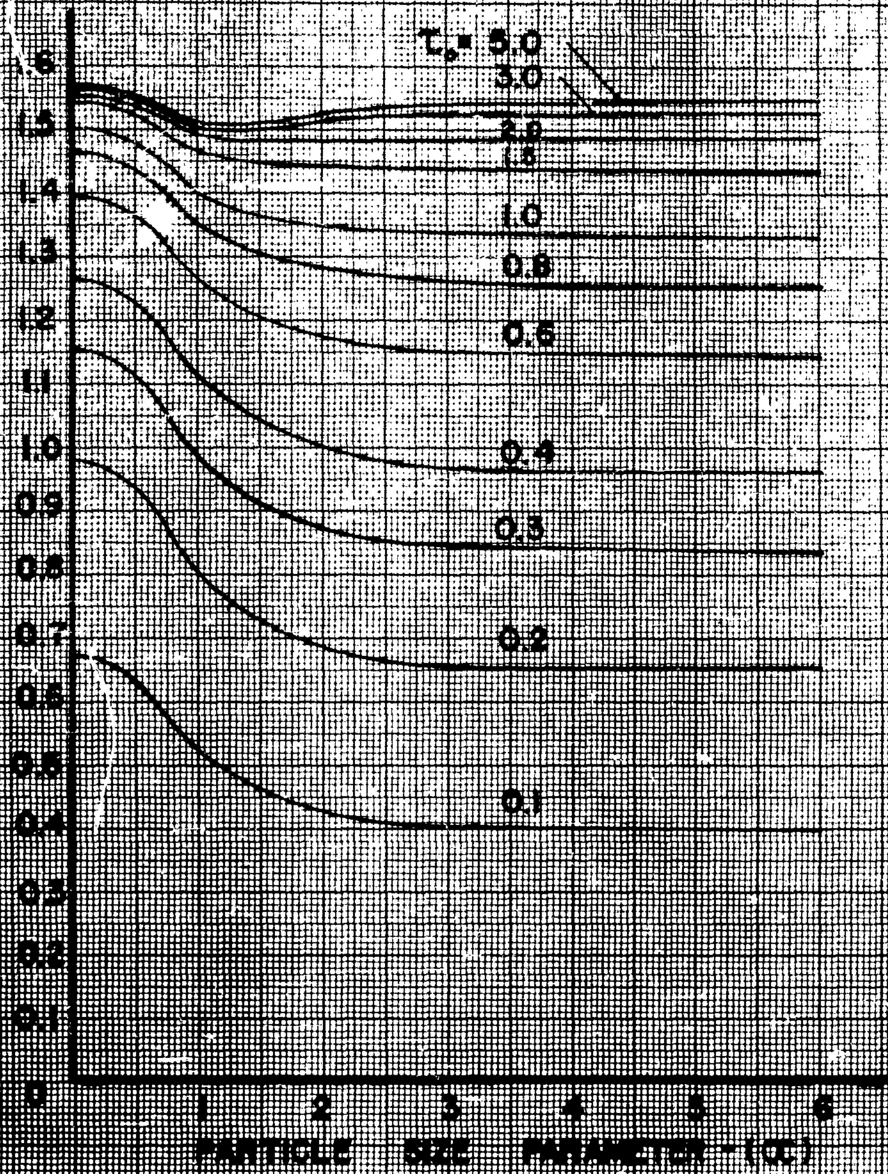


VALUES OF PARAMETERS

M & N

$\rho_1 = 0.5$ $\rho_2 = 0.9$ $m = 1.25 - 1.25i$

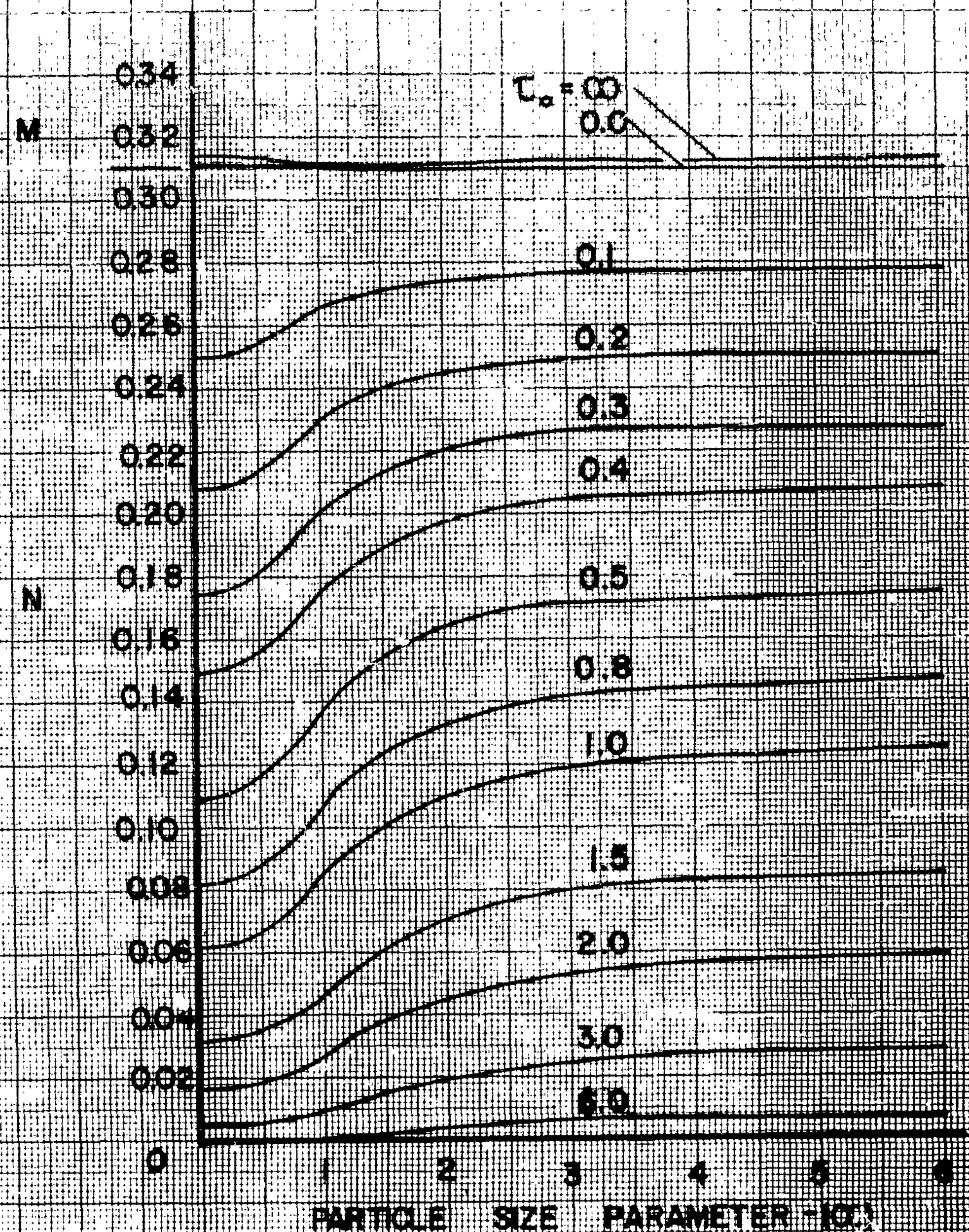
FIGURE 21



VALUES OF PARAMETER Q

$\rho_1 = 0.5$ $\rho_2 = 0.9$ $m = 1.25 - 1.25i$

FIGURE 22

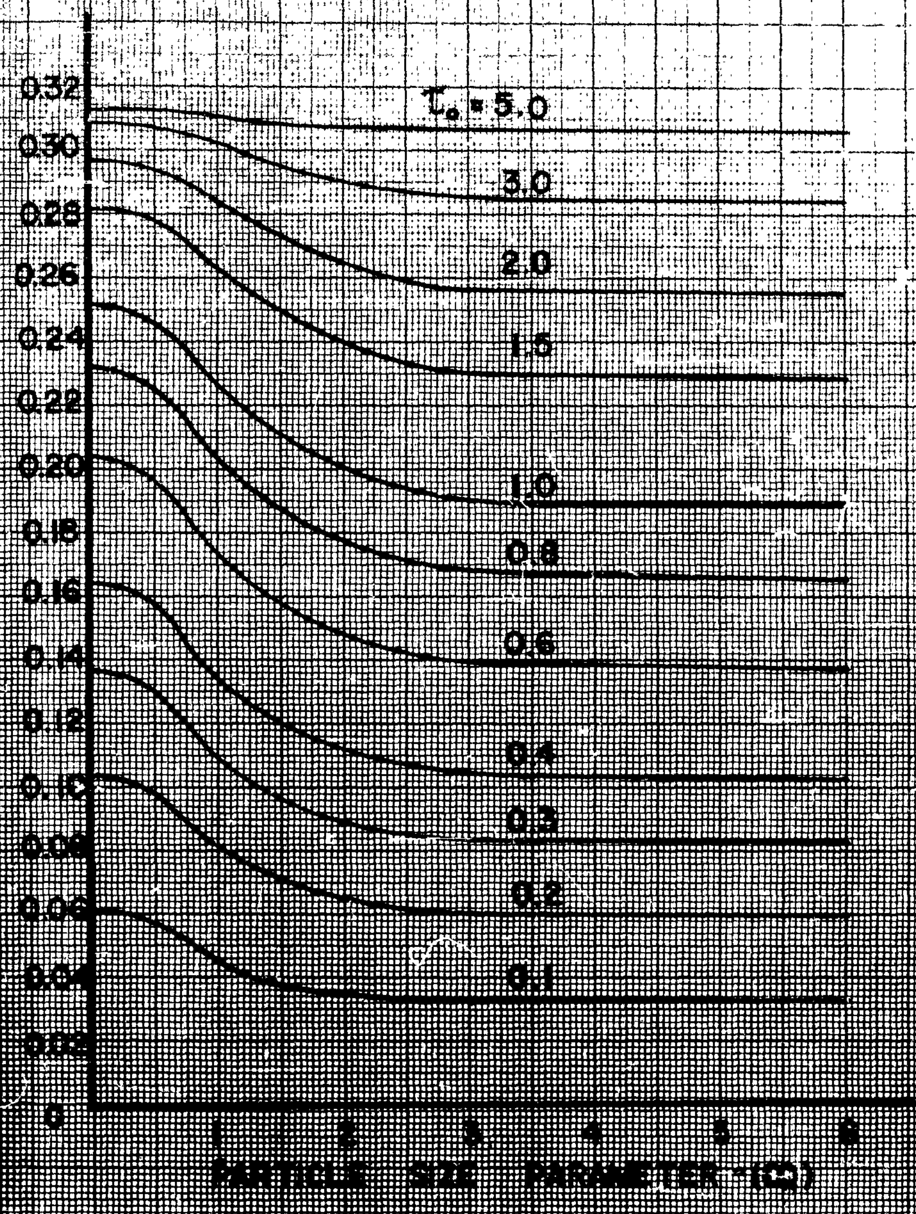


VALUES OF PARAMETERS

M & N

$\rho_1 = 0.9$ $\rho_2 = 0.1$ $m = 1.25 - 1.25i$

FIGURE 23

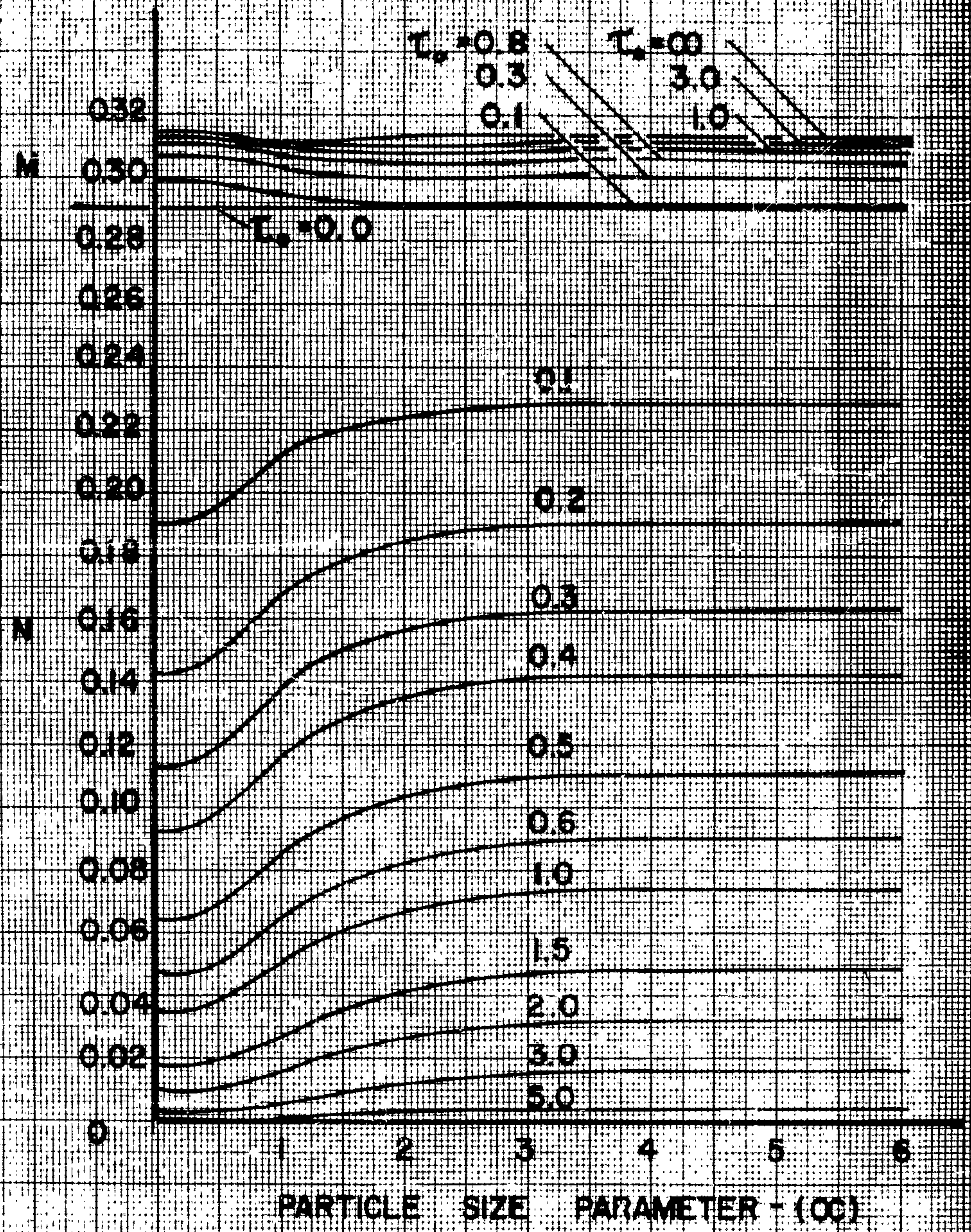


VALUES OF PARAMETER

Q

$\rho_1 = 0.9$ $\rho_2 = 0.1$ $m = 1.25 - 1.25$

FIGURE 24

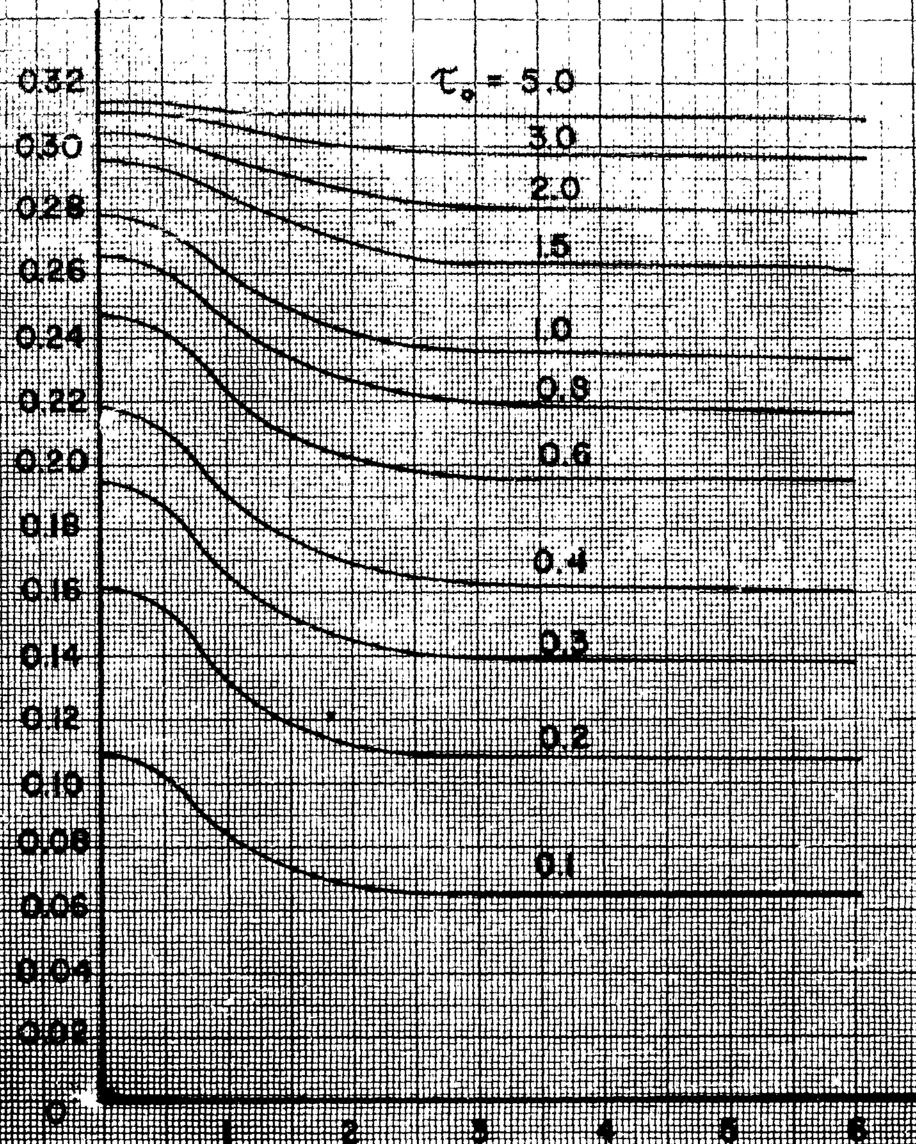


VALUES OF PARAMETERS

M & N

$\rho_1 = 0.9 \qquad \rho_2 = 0.5 \qquad m = 1.25 - 1.25i$

FIGURE 25

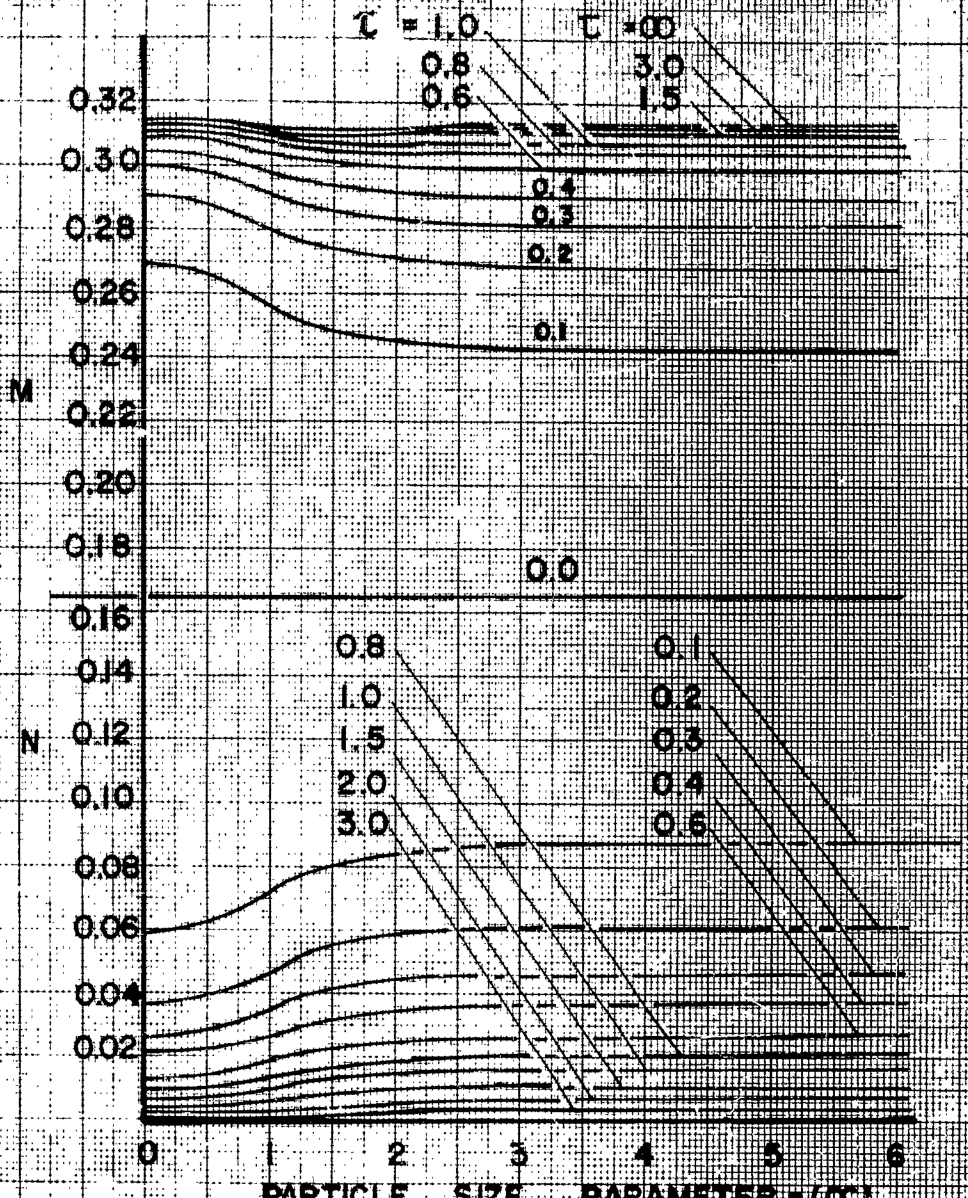


PARTICLE SIZE PARAMETER - (α)

VALUES OF PARAMETER Q

ρ_1 = 0.9 ρ_2 = 0.5 m = 1.25 - 1.25

FIGURE 26

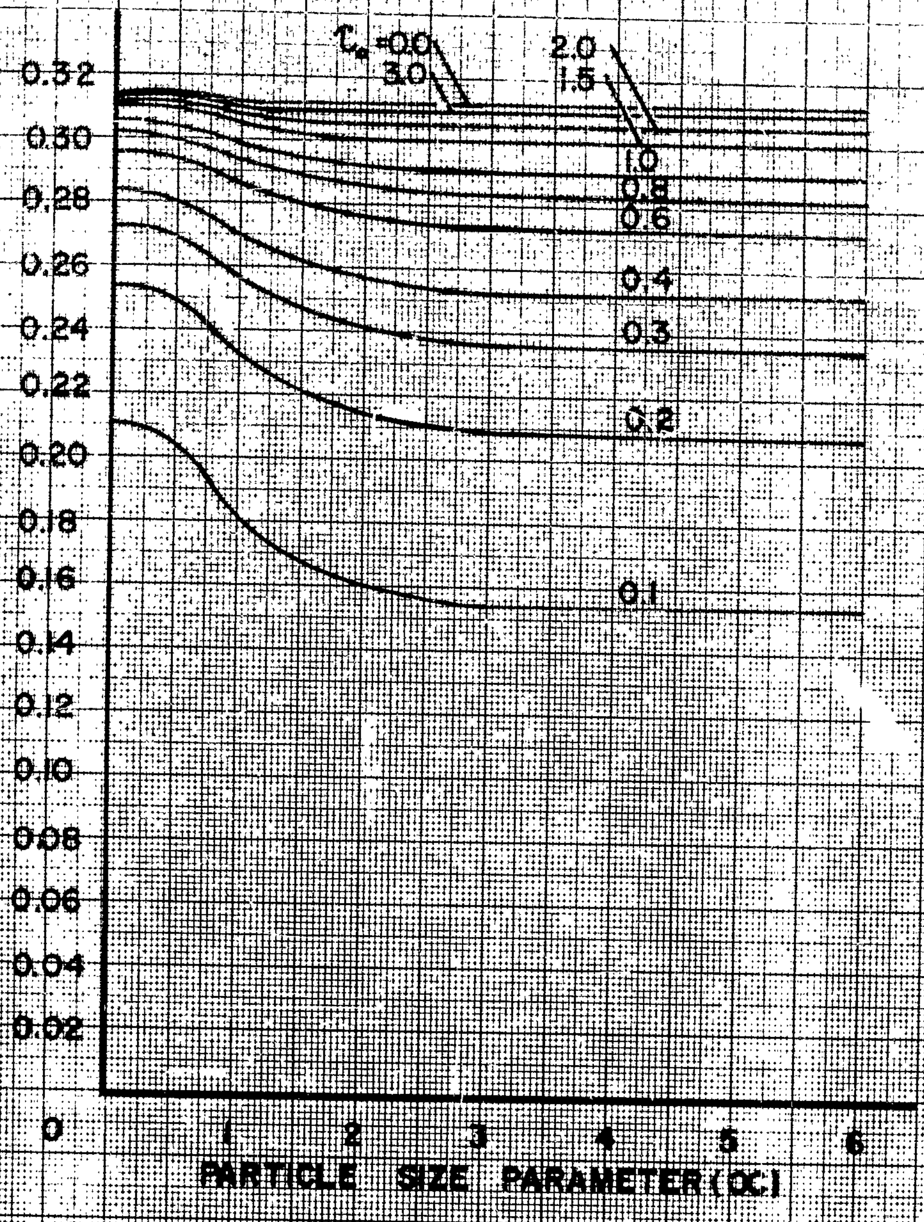


VALUES OF PARAMETER

M & N

$\rho_1 = 0.9$ $\rho_2 = 0.9$ $m = 1.25 - 1.25i$

FIGURE 27



VALUES OF PARAMETER

Q

$\beta_1 = 0.9$ $\beta_2 = 0.9$

$m = 1.25 - .125i$

FIGURE 29

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.3

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3118	0.2517	0.0601
0.2	0.3124	0.2096	0.1029
0.3	0.3129	0.1769	0.1360
0.4	0.3132	0.1506	0.1626
0.6	0.3136	0.1111	0.2025
0.8	0.3138	0.0833	0.2305
1.0	0.3139	0.0632	0.2507
1.5	0.3140	0.0327	0.2813
3.0	0.3140	0.0052	0.3088
5.0	0.3140	0.0005	0.3135
	0.3140	0.0	0.3140

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.3

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2982	0.1909	0.1073
0.2	0.3039	0.1436	0.1603
0.3	0.3072	0.1139	0.1933
0.4	0.3092	0.0930	0.2162
0.6	0.3115	0.0654	0.2461
0.8	0.3126	0.0478	0.1457
1.0	0.3132	0.0358	0.2774
1.5	0.3138	0.0183	0.2955
3.0	0.3140	0.0029	0.3111
5.0	0.3140	0.0003	0.3137
	0.3140	0.0	0.3140

Refractive index of scattering medium = 1.25-1.25 i

Particle size parameter α = 0.3

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2690	0.0602	0.2088
0.2	0.2902	0.0375	0.2527
0.3	0.2993	0.0271	0.2723
0.4	0.3043	0.0209	0.2833
0.6	0.3092	0.0139	0.2953
0.8	0.3114	0.0099	0.3015
1.0	0.3126	0.0073	0.3053
1.5	0.3136	0.0037	0.3100
3.0	0.3140	0.0006	0.3134
5.0	0.3140	0.0001	0.3140
	0.3140	0.0	0.3140

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6235	2.1577	0.4658
0.2	2.6593	1.8403	0.8191
0.3	2.7848	1.5813	1.1034
0.4	2.7033	1.3653	1.3380
0.6	2.7269	1.0282	1.6987
0.8	2.7401	0.7828	1.9574
1.0	2.7476	0.6009	2.1467
1.5	2.7554	0.3183	2.4371
3.0	2.7584	0.0530	2.7054
5.0	2.7585	0.0055	2.7530
	2.7585	0.0	2.7585

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8674	1.2384	0.6290
0.2	2.1126	1.0501	0.4674
0.3	2.2827	0.8987	1.3841
0.4	2.4045	0.7736	1.6309
0.6	2.5581	0.5804	1.9777
0.8	2.6425	0.4409	2.2016
1.0	2.6901	0.3381	2.3521
1.5	2.7393	0.1789	1.3694
3.0	2.7579	0.0298	2.7282
5.0	2.7585	0.0006	2.757[illegible]
	2.7585	0.0	2.7585

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.0596	0.2562	0.8034
0.2	1.5353	0.2159	1.3194
0.3	1.8618	0.1839	1.6778
0.4	2.0935	0.1579	1.9357
0.6	2.3838	0.1180	2.2648
0.8	2.5421	0.0894	2.4526
1.0	2.6312	0.0685	2.5627
1.5	2.7228	0.0362	2.6866
3.0	2.7575	0.0060	2.7515
5.0	2.7585	0.0006	2.7578
	2.7585	0.0	2.7585

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5058	1.2384	0.2673
0.2	1.5175	1.0501	0.4674
0.3	1.5258	0.8987	0.6271
0.4	1.5317	0.7736	0.7581
0.6	1.5393	0.5804	0.9589
0.8	1.5435	0.4409	1.1026
1.0	1.5458	0.3381	1.2078
1.5	1.5483	0.1789	1.3694
3.0	1.5493	0.0298	1.5195
5.0	1.5493	0.0031	1.5462
	1.5493	0.0	1.5493

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2218	0.8103	0.4116
0.2	1.3222	0.6573	0.6650
0.3	1.3869	0.5460	0.8409
0.4	1.4310	0.4604	0.9706
0.6	1.4840	0.3367	1.1473
0.8	1.5120	0.2523	1.2597
1.0	1.5275	0.1920	1.3355
1.5	1.5432	0.1008	1.4425
3.0	1.5491	0.0167	1.5324
5.0	1.5493	0.0017	1.5475
	1.5493	0.0	1.5493

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8152	0.1971	0.6181
0.2	1.0703	0.1505	0.9198
0.3	1.2194	0.1205	1.0989
0.4	1.3147	0.0991	1.2156
0.6	1.5236	0.0705	1.3531
0.8	1.4786	0.0520	1.4266
1.0	1.5083	0.0393	1.4690
1.5	1.5380	0.0204	1.5175
3.0	1.5490	0.0034	1.5456
5.0	1.5493	0.0004	1.5489
	1.5493	0.0	1.5493

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3115	0.2562	0.0553
0.2	0.3120	0.2159	0.0961
0.3	0.3123	0.1840	0.1284
0.4	0.3126	0.1579	0.1547
0.6	0.3129	0.1180	0.1949
0.8	0.3131	0.0894	0.2236
1.0	0.3131	0.0685	0.2447
1.5	0.3133	0.0362	0.2771
3.0	0.3133	0.0060	0.3073
5.0	0.3133	0.0006	0.3127
	0.3133	0.0	0.3133

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2972	0.1971	0.1001
0.2	0.3028	0.1505	0.1523
0.3	0.3060	0.1205	0.1856
0.4	0.3081	0.0991	0.2090
0.6	0.3105	0.0705	0.2401
0.8	0.3117	0.0520	0.2597
1.0	0.3124	0.0393	0.2731
1.5	0.3130	0.0204	0.2926
3.0	0.3133	0.0034	0.3099
5.0	0.3133	0.0004	0.3129
	0.3133	0.0	0.3133

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2650	0.0641	0.2010
0.2	0.2873	0.0404	0.2469
0.3	0.2970	0.0293	0.2677
0.4	0.3124	0.0228	0.2796
0.6	0.3978	0.0152	0.2926
0.8	0.3103	0.0109	0.2994
1.0	0.3116	0.0081	0.3035
1.5	0.3128	0.0042	0.3087
3.0	0.3133	0.0007	0.3126
5.0	0.3133	0.0001	0.3132
	0.3133	0.0	0.3133

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5854	2.2108	0.3746
0.2	2.5994	1.9248	0.6745
0.3	2.6108	1.6854	0.9254
0.4	2.6199	1.4811	1.1389
0.6	2.6329	1.1527	1.4802
0.8	2.6410	0.9044	1.7366
1.0	2.6461	0.7142	1.9321
1.5	2.6521	0.4031	2.2490
3.0	2.6549	0.0789	2.5761
5.0	2.6550	0.0098	2.6453
	2.6550	0.0	2.6550

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.9222	0.2659	0.6563
0.2	1.3447	0.2304	1.1143
0.3	1.6526	0.2010	1.4516
0.4	1.8822	0.1760	1.7062
0.6	2.1880	0.1364	2.0516
0.8	2.3679	0.1067	2.2611
1.0	2.4761	0.0841	2.3920
1.5	2.5980	0.0474	2.5506
3.0	2.6529	0.0093	2.6436
5.0	2.6550	0.0011	2.6539
	2.6550	0.0	2.6550

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7867	1.2768	0.5099
0.2	1.9953	1.1091	0.8862
0.3	2.1486	0.9693	1.1792
0.4	2.2635	0.8506	1.4130
0.6	2.4175	0.6606	1.7569
0.8	2.5086	0.5176	1.9901
1.0	2.5636	0.4083	2.1553
1.5	2.6258	0.2303	2.3955
3.0	2.6539	0.0450	2.6089
5.0	2.6550	0.0011	2.6539
	2.6550	0.0	2.6550

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.2931	1.2768	0.2163
0.2	1.4978	1.1091	0.3887
0.3	1.5016	0.9693	0.5322
0.4	1.5046	0.8506	1.4130
0.6	1.5089	0.6606	1.7569
0.8	1.5115	0.5176	0.9939
1.0	1.5132	0.4083	1.1049
1.5	1.5151	0.2303	1.2848
3.0	1.5161	0.0450	1.4710
5.0	1.5161	0.0056	1.5105
	1.5161	0.0	1.5161

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1868	0.8481	0.3387
0.2	1.2753	0.7089	0.5664
0.3	1.3362	0.6029	0.7334
0.4	1.3798	0.5185	0.8613
0.6	1.4355	0.3923	1.0433
0.8	1.4672	0.3027	1.1645
1.0	1.4858	0.2367	1.2492
1.5	1.5065	0.1321	1.3744
3.0	1.5157	0.0257	1.4900
5.0	1.5161	0.0032	1.5129
	1.5161	0.0	1.5161

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7314	0.2109	0.5205
0.2	0.9741	0.1669	0.8072
0.3	1.1260	0.1369	0.9891
0.4	1.2282	0.1149	1.1133
0.6	1.3514	0.0843	1.2671
0.8	1.4179	0.0639	1.3540
1.0	1.4560	0.0495	1.4066
1.5	1.4973	0.0273	1.4700
3.0	1.5154	0.0053	1.5101
5.0	1.5161	0.0007	1.5154
	1.5161	0.0	1.5161

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3109	0.2659	0.0450
0.2	0.3111	0.2304	0.0807
0.3	0.3113	0.2010	0.1103
0.4	0.3114	0.1760	0.1354
0.6	0.3116	0.1364	0.1752
0.8	0.3117	0.1067	0.2050
1.0	0.3118	0.0841	0.2277
1.5	0.3119	0.0474	0.2645
3.0	0.3119	0.0093	0.3026
5.0	0.3119	0.0011	0.3108
	0.3119	0.0	0.3119

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2951	0.2109	0.0842
0.2	0.3002	0.1669	0.1334
0.3	0.3035	0.1369	0.1666
0.4	0.3057	0.1149	0.1908
0.6	0.3083	0.0843	0.2241
0.8	0.3098	0.0639	0.2459
1.0	0.3106	0.0495	0.2611
1.5	0.3115	0.0273	0.2842
3.0	0.3119	0.0053	0.3066
5.0	0.3119	0.0007	0.3113
	0.3119	0.0	0.3119

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2555	0.0737	0.1818
0.2	0.2799	0.0480	0.2319
0.3	0.2912	0.0354	0.2558
0.4	0.2976	0.0278	0.2697
0.6	0.3043	0.0190	0.2853
0.8	0.3075	0.0139	0.2937
1.0	0.3093	0.0105	0.2988
1.5	0.3111	0.0057	0.3054
3.0	0.3119	0.0011	0.3108
5.0	0.3119	0.0001	0.3118
	0.3119	0.0	0.3119

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5828	2.2849	0.2980
0.2	2.5987	2.0510	0.5477
0.3	2.6134	1.8494	0.7640
0.4	2.6261	1.6723	0.9538
0.6	2.6463	1.3758	1.2705
0.8	2.6606	1.1389	1.5218
1.0	2.6708	0.9472	1.7237
1.5	2.6851	0.6059	2.0792
3.0	2.6948	0.1680	2.5269
5.0	2.6957	0.0319	2.6637
	2.6957	0.0	2.6957

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7294	1.3201	0.4092
0.2	1.9128	1.1819	0.7309
0.3	2.0570	1.0632	0.9938
0.4	2.1722	0.9594	1.2127
0.6	2.3400	0.7867	1.5533
0.8	2.4513	0.6497	1.8015
1.0	2.5263	0.5395	1.9868
1.5	2.6261	0.3443	2.2818
3.0	2.6903	0.0953	2.5950
5.0	2.6955	0.0181	2.6774
	2.6957	0.0	2.6957

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8048	0.2750	0.5298
0.2	1.1738	0.2455	0.9283
0.3	1.4606	0.2203	1.2403
0.4	1.6875	0.1984	1.4892
0.6	2.0153	0.1621	1.8532
0.8	2.2303	0.1335	2.0968
1.0	2.3742	0.1107	2.2635
1.5	2.5643	0.0705	2.4938
3.0	2.6856	0.0195	2.6661
5.0	2.6953	0.0037	2.6916
	2.6957	0.0	2.6957

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4923	1.3201	0.1722
0.2	1.4976	1.1819	0.3156
0.3	1.5024	1.0632	0.4392
0.4	1.5066	0.9594	0.5472
0.6	1.5132	0.7867	0.7265
0.8	1.5179	0.6497	0.8682
1.0	1.5212	0.5395	0.9818
1.5	1.5259	0.3443	1.1816
3.0	1.5290	0.0953	1.4337
5.0	1.5293	0.0181	1.5111
	1.5293	0.0	1.5293

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1612	0.8864	0.2748
0.2	1.2411	0.7669	0.4742
0.3	1.3003	0.6720	0.6282
0.4	1.3453	0.5942	0.7511
0.6	1.4079	0.4733	0.9345
0.8	1.4474	0.3836	1.0638
1.0	1.4732	0.3146	1.1586
1.5	1.5066	0.1975	1.3091
3.0	1.5275	0.0541	1.4734
5.0	1.5292	0.0103	1.5189
	1.5293	0.0	1.5293

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6555	0.2240	0.4315
0.2	0.8812	0.1843	0.6968
0.3	1.0335	0.1559	0.8776
0.4	1.1422	0.1343	1.0079
0.6	1.2834	0.1032	1.1802
0.8	1.3674	0.0819	1.2855
1.0	1.4202	0.0662	1.3540
1.5	1.4861	0.0408	1.4452
3.0	1.5260	0.0111	1.5150
5.0	1.5292	0.0021	1.5271
	1.5293	0.0	1.5293

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6555	0.2240	0.4315
0.2	0.8812	0.1843	0.6968
0.3	1.0335	0.1559	0.8776
0.4	1.1422	0.1343	1.0079
0.6	1.2834	0.1032	1.1802
0.8	1.3674	0.0819	1.2855
1.0	1.4202	0.0662	1.3540
1.5	1.4861	0.0408	1.4452
3.0	1.5260	0.0111	1.5150
5.0	1.5292	0.0021	1.5271
	1.5293	0.0	1.5293

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3109	0.2750	0.0359
0.2	0.3111	0.2455	0.0656
0.3	0.3113	0.2203	0.0910
0.4	0.3115	0.1984	0.1131
0.6	0.3118	0.1621	0.1497
0.8	0.3120	0.1335	0.1784
1.0	0.3121	0.1107	0.2014
1.5	0.3123	0.0705	0.2418
3.0	0.3125	0.0195	0.2930
5.0	0.3125	0.0037	0.3088
	0.3125	0.0	0.3125

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2935	0.2240	0.0694
0.2	0.2983	0.1843	0.1140
0.3	0.3016	0.1559	0.1457
0.4	0.3040	0.1343	0.1697
0.6	0.3071	0.1032	0.2038
0.8	0.3089	0.0819	0.2270
1.0	0.3101	0.0662	0.2438
1.5	0.3115	0.0408	0.2707
3.0	0.3124	0.0111	0.3013
5.0	0.3125	0.0037	0.3088
	0.3125	0.0	0.3125

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2456	0.0839	0.1617
0.2	0.2716	0.0568	0.2148
0.3	0.2846	0.0429	0.2416
0.4	0.2922	0.0344	0.2579
0.6	0.3007	0.0242	0.2765
0.8	0.3051	0.0183	0.2868
1.0	0.3076	0.0143	0.2933
1.5	0.3106	0.0085	0.3021
3.0	0.3123	0.0023	0.3101
5.0	0.3125	0.0004	0.3120
	0.3125	0.0	0.3125

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5929	2.3181	0.2748
0.2	2.6159	2.1082	0.5077
0.3	2.6361	1.9250	0.7111
0.4	2.6533	1.7620	0.8913
0.6	2.6802	1.4842	1.1959
0.8	2.6994	1.2571	1.4423
1.0	2.7133	1.0693	1.6440
1.5	2.7335	0.7233	2.0102
3.0	2.7492	0.2376	2.5116
5.0	2.7510	0.0571	2.6939
	2.7512	0.0	2.7512

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7159	1.3371	0.3788
0.2	1.8932	1.2115	0.6817
0.3	2.0356	1.1026	0.9330
0.4	2.1515	1.0065	1.1451
0.6	2.3257	0.8441	1.4816
0.8	2.4459	0.7127	1.7332
1.0	2.5303	0.6049	1.9253
1.5	2.6501	0.4079	2.2422
3.0	2.7402	0.1337	2.6066
5.0	2.7505	0.0321	2.7184
	2.7512	0.0	2.7512

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7691	0.2781	0.4910
0.2	1.1191	0.2509	0.8682
0.3	1.3966	0.2276	1.1690
0.4	1.6207	0.2071	1.4136
0.6	1.9540	0.1729	1.7811
0.8	2.1818	0.1455	2.0362
1.0	2.3404	0.1232	2.2172
1.5	2.5641	0.0828	2.4812
3.0	2.7310	0.0271	2.7039
5.0	2.7500	0.0065	2.7435
	2.7512	0.0	2.7512

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4956	1.3371	0.1585
0.2	1.5033	1.2115	0.2918
0.3	1.5099	1.1026	0.4073
0.4	1.5155	1.0065	0.5091
0.6	1.5243	0.8441	0.6802
0.8	1.5305	0.7127	0.8177
1.0	1.5349	0.6049	0.9300
1.5	1.5414	0.4079	1.1335
3.0	1.5464	0.1337	1.4127
5.0	1.5469	0.0321	1.5148
	1.5470	0.0	1.5470

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1551	0.9001	0.2550
0.2	1.2328	0.7889	0.4439
0.3	1.2916	0.6996	0.5920
0.4	1.3374	0.6256	0.7118
0.6	1.4027	0.5091	0.8936
0.8	1.4455	0.4212	1.0243
1.0	1.4746	0.3525	1.1220
1.5	1.5145	0.2331	1.2814
3.0	1.5435	0.0753	1.4682
5.0	1.5468	0.0181	1.5287
	1.5470	0.0	1.5470

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6317	0.2284	0.4033
0.2	0.8500	0.1906	0.6594
0.3	1.0010	0.1631	0.8379
0.4	1.1112	0.1420	0.9692
0.6	1.2583	0.1114	1.1470
0.8	1.3490	0.0900	1.2590
1.0	1.4080	0.0741	1.3339
1.5	1.4860	0.0480	1.4380
3.0	1.5406	0.0153	1.5253
5.0	1.5466	0.0037	1.5429
	1.5470	0.0	1.5470

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3110	0.2781	0.0330
0.2	0.3114	0.2509	0.0604
0.3	0.3116	0.2276	0.0841
0.4	0.3119	0.2071	0.1048
0.6	0.3123	0.1729	0.1393
0.8	0.3125	0.1455	0.1670
1.0	0.3127	0.1232	0.1895
1.5	0.3130	0.0828	0.2302
3.0	0.3132	0.0271	0.2861
5.0	0.3132	0.0065	0.3067
	0.3132	0.0	0.3132

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2931	0.2284	0.0647
0.2	0.2978	0.1906	0.1072
0.3	0.3011	0.1631	0.1380
0.4	0.3036	0.1420	0.1616
0.6	0.3068	0.1114	0.1955
0.8	0.3088	0.0900	0.2188
1.0	0.3101	0.0741	0.2360
1.5	0.3118	0.0480	0.2638
3.0	0.3131	0.0153	0.2978
5.0	0.3132	0.0037	0.3095
	0.3132	0.0	0.3132

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2422	0.0875	0.1546
0.2	0.2686	0.0602	0.2084
0.3	0.2821	0.0460	0.2361
0.4	0.2902	0.0371	0.2531
0.6	0.2993	0.0265	0.2728
0.8	0.3042	0.0203	0.2839
1.0	0.3071	0.0162	0.2909
1.5	0.3106	0.0100	0.3006
3.0	0.3129	0.0031	0.3098
5.0	0.3132	0.0007	0.3124
	0.3132	0.0000	0.3132

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5968	2.3232	0.2735
0.2	2.6222	2.1169	0.5053
0.3	2.6440	1.9363	0.7077
0.4	2.6625	1.7754	0.8871
0.6	2.6913	1.5001	1.1911
0.8	2.7118	1.2743	1.4375
1.0	2.7265	1.0870	1.6396
1.5	2.7481	0.7403	2.0079
3.0	2.7651	0.2484	2.5167
5.0	2.7672	0.0615	2.7057
	2.7673	0.0	2.7673

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7164	1.3392	0.3772
0.2	1.8943	1.2153	0.6790
0.3	2.0372	1.1077	0.9295
0.4	2.1539	1.0126	1.1413
0.6	2.3298	0.8516	1.4781
0.8	2.4518	0.7210	1.7307
1.0	2.5378	0.6136	1.9242
1.5	2.6609	0.4165	2.2444
3.0	2.7553	0.1394	2.6159
5.0	2.7666	0.0345	2.7321
	2.7673	0.0	2.7673

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7672	0.2783	0.4889
0.2	1.1161	0.2514	0.8647
0.3	1.3931	0.2283	1.1648
0.4	1.6174	0.2081	1.4093
0.6	1.9521	0.1741	1.7780
0.8	2.1820	0.1469	2.0351
1.0	2.3429	0.1247	2.2182
1.5	2.5714	0.0844	2.4871
3.0	2.7453	0.0282	2.7172
5.0	2.7660	0.0070	2.7590
	2.7673	0.0	2.7673

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4969	1.3392	0.1577
0.2	1.5053	1.2153	0.2901
0.3	1.5125	1.1077	0.4048
0.4	1.5185	1.0126	0.5060
0.6	1.5278	0.8516	0.6762
0.8	1.5344	0.7210	0.8134
1.0	1.5392	0.6136	0.9256
1.5	1.5460	0.4164	1.1296
3.0	1.5514	0.1394	1.4120
5.0	1.5520	0.0345	1.5175
	1.5521	0.0	1.5521

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1553	0.9015	0.2539
0.2	1.2333	0.7912	0.4421
0.3	1.2923	0.7027	0.5896
0.4	1.3383	0.6292	0.7091
0.6	1.4042	0.5133	0.8909
0.8	1.4476	0.4257	1.0219
1.0	1.4771	0.3571	1.1200
1.5	1.5180	0.2376	1.2804
3.0	1.5483	0.0783	1.4700
5.0	1.5518	0.0194	1.5325
	1.5521	0.0	1.5521

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6304	0.2287	0.4017
0.2	0.8482	0.1911	0.6572
0.3	0.9993	0.1638	0.8355
0.4	1.1096	0.1427	0.9669
0.6	1.2575	0.1122	1.1453
0.8	1.3491	0.0908	1.2582
1.0	1.4089	0.0750	1.3339
1.5	1.4885	0.0488	1.4396
3.0	1.5451	0.0158	1.5293
5.0	1.5516	0.0039	1.5477
	1.5521	0.0	1.5521

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3111	0.2783	0.0328
0.2	0.3115	0.2514	0.0600
0.3	0.3118	0.2283	0.0834
0.4	0.3120	0.2081	0.1040
0.6	0.3124	0.1741	0.1383
0.8	0.3127	0.1469	0.1657
1.0	0.3129	0.1247	0.1881
1.5	0.3132	0.0844	0.2288
3.0	0.3134	0.0282	0.2852
5.0	0.3134	0.0070	0.3064
	0.3134	0.0	0.3134

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2931	0.2287	0.0644
0.2	0.2979	0.1911	0.1068
0.3	0.3012	0.1638	0.1374
0.4	0.3036	0.1427	0.1609
0.6	0.3069	0.1122	0.1947
0.8	0.3089	0.0908	0.2181
1.0	0.3102	0.0750	0.2352
1.5	0.3120	0.0488	0.2632
3.0	0.3132	0.0158	0.2974
5.0	0.3134	0.0039	0.3095
	0.3134	0.0	0.3134

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 5.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2420	0.0878	0.1542
0.2	0.2684	0.0605	0.2080
0.3	0.2819	0.0462	0.2357
0.4	0.2900	0.0373	0.2427
0.6	0.2993	0.0267	0.2726
0.8	0.3042	0.0205	0.2837
1.0	0.3071	0.0163	0.2908
1.5	0.3107	0.0102	0.3005
3.0	0.3131	0.0032	0.3099
5.0	0.3134	0.0008	0.3126
	0.3134	0.0	0.3134

Refractive index of scattering medium 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5986	2.3250	0.2736
0.2	2.6247	2.1198	0.5049
0.3	2.6468	1.9401	0.7067
0.4	2.6654	1.7798	0.8856
0.6	2.6941	1.5055	1.1886
0.8	2.7144	1.2801	1.4343
1.0	2.7290	1.0932	1.6359
1.5	2.7503	0.7469	2.0034
3.0	2.7671	0.2540	2.5131
5.0	2.7692	0.0643	2.7049
	2.7693	0.0	2.7693

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7172	1.3399	0.3773
0.2	1.8951	1.2164	0.6786
0.3	2.0378	1.1093	0.9285
0.4	2.1544	1.0146	1.1398
0.6	2.3302	0.8543	1.4759
0.8	2.4522	0.7241	1.7281
1.0	2.5382	0.6169	1.9214
1.5	2.6616	0.4200	2.2415
3.0	2.7569	0.1425	2.6144
5.0	2.7685	0.0360	2.7325
	2.7693	0.0	2.7693

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7675	0.2784	0.4891
0.2	1.1158	0.2516	0.8642
0.3	1.3922	0.2285	1.1637
0.4	1.6160	0.2084	1.4076
0.6	1.9503	0.1746	1.7757
0.8	2.1802	0.1475	2.0328
1.0	2.3413	0.1253	2.2160
1.5	2.5706	0.0851	2.4855
3.0	2.7464	0.0288	2.7177
5.0	2.7679	0.0073	2.7606
	2.7693	0.0	2.7693

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter $\alpha = 6.0$

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4975	1.3399	0.1577
0.2	1.5062	1.2164	0.2897
0.3	1.5134	1.1093	0.4041
0.4	1.5195	1.0146	0.5048
0.6	1.5287	0.8543	0.6745
0.8	1.5353	0.7241	0.8112
1.0	1.5399	0.6169	0.9231
1.5	1.5467	0.4200	1.1267
3.0	1.5520	0.1425	1.4095
5.0	1.5527	0.0360	1.5166
	1.5527	0.0	1.5527

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1557	0.9017	0.2539
0.2	1.2336	0.7919	0.4418
0.3	1.2926	0.7036	0.5889
0.4	1.3385	0.6304	0.7081
0.6	1.4043	0.5148	0.8895
0.8	1.4477	0.4275	1.0202
1.0	1.4773	0.3590	1.1183
1.5	1.5182	0.2396	1.2786
3.0	1.5488	0.0800	1.4687
5.0	1.5525	0.0202	1.5322
	1.5527	0.0	1.5527

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6305	0.2287	0.4018
0.2	0.8481	0.1912	0.6569
0.3	0.9988	0.1640	0.8348
0.4	1.1090	0.1430	0.9660
0.6	1.2568	0.1125	1.1443
0.8	1.3484	0.0912	1.2572
1.0	1.4084	0.0754	1.3330
1.5	1.4882	0.0492	1.4390
3.0	1.5455	0.0162	1.5293
5.0	1.5522	0.0041	1.5482
	1.5527	0.0	1.5527

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3111	0.2784	0.0328
0.2	0.3115	0.2516	0.0599
0.3	0.3118	0.2285	0.0833
0.4	0.3121	0.2084	0.1037
0.6	0.3124	0.1746	0.1378
0.8	0.3127	0.1475	0.1652
1.0	0.3129	0.1253	0.1876
1.5	0.3132	0.0850	0.2281
3.0	0.3134	0.0288	0.2846
5.0	0.3134	0.0073	0.3062
	0.3134	0.0	0.3134

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2931	0.2287	0.0644
0.2	0.2979	0.1912	0.1067
0.3	0.3012	0.1640	0.1372
0.4	0.3036	0.1430	0.1606
0.6	0.3069	0.1125	0.1944
0.8	0.3089	0.0912	0.2177
1.0	0.3102	0.0754	0.2348
1.5	0.3120	0.0492	0.2628
3.0	0.3133	0.0162	0.2971
5.0	0.3134	0.0041	0.3093
	0.3134	0.0	0.3134

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2420	0.0878	0.1542
0.2	0.2684	0.0605	0.2079
0.3	0.2819	0.0463	0.2356
0.4	0.2900	0.0374	0.2526
0.6	0.2992	0.0268	0.2724
0.8	0.3041	0.0206	0.2836
1.0	0.3071	0.0164	0.2906
1.5	0.3107	0.0103	0.3004
3.0	0.3131	0.0033	0.3099
5.0	0.3134	0.0008	0.3126
	0.3134	0.0	0.3134

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6152	2.1720	0.4431
0.2	2.6468	1.8630	0.7838
0.3	2.6699	1.6091	1.0607
0.4	2.6869	1.3960	1.2909
0.6	2.7092	1.0609	1.6482
0.8	2.7219	0.8144	1.9075
1.0	2.7293	0.6301	2.0993
1.5	2.7373	0.3397	2.3976
3.0	2.7405	0.0591	2.6814
5.0	2.7406	0.0064	2.7342
	2.7406	0.0	2.7406

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8480	1.2483	0.5996
0.2	2.0853	1.0652	1.0201
0.3	2.2525	0.9167	1.3358
0.4	2.3736	0.7931	1.5805
0.6	2.5289	0.6006	1.9283
0.8	2.6159	0.4701	2.1558
1.0	2.6660	0.3555	2.3104
1.5	2.7189	0.1914	2.5275
3.0	2.7400	0.0333	2.7067
5.0	2.7406	0.0036	2.7370
	2.7406	0.0	2.7406

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.0259	0.2586	0.7673
0.2	1.4900	0.2195	1.2705
0.3	1.8134	0.1881	1.6253
0.4	2.0459	0.1623	1.8836
0.6	2.3417	0.1224	2.2192
0.8	2.5062	0.0936	2.4126
1.0	2.6006	0.0722	2.5283
1.5	2.7000	0.0388	2.6611
3.0	2.7394	0.0068	2.7326
5.0	2.7406	0.0007	2.7399
	2.7406	0.0	2.7406

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5030	1.2483	0.2547
0.2	1.5134	1.0652	0.4482
0.3	1.5209	0.9167	0.6043
0.4	1.5264	0.7931	0.7334
0.6	1.5336	0.6006	0.9330
0.8	1.5377	0.4601	1.0776
1.0	1.5400	0.3555	1.1845
1.5	1.5426	0.1914	1.3511
3.0	1.5436	0.0333	1.5103
5.0	1.5436	0.0036	1.5400
	1.5436	0.0	1.5436

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2135	0.8197	0.3938
0.2	1.3115	0.6700	0.6415
0.3	1.3757	0.5599	0.8159
0.4	1.4200	0.4745	0.9455
0.6	1.4741	0.3501	1.1240
0.8	1.5033	0.2644	1.2389
1.0	1.5197	0.2027	1.3170
1.5	1.5367	0.1082	1.4285
3.0	1.5434	0.0188	1.5247
5.0	1.5436	0.0020	1.5416
	1.5436	0.0	1.5436

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7951	0.2004	0.5947
0.2	1.0481	0.1544	0.8937
0.3	1.1985	0.1243	1.0742
0.4	1.2958	0.1028	1.1930
0.6	1.4085	0.0736	1.3348
0.8	1.4664	0.0548	1.4116
1.0	1.4982	0.0416	1.4566
1.5	1.5307	0.0220	1.5086
3.0	1.5432	0.0038	1.5394
5.0	1.5436	0.0004	1.5432
	1.5436	0.0	1.5436

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.7

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3114	0.2546	0.0528
0.2	0.3118	0.2195	0.0923
0.3	0.3121	0.1881	0.1240
0.4	0.3123	0.1623	0.1501
0.6	0.3126	0.1224	0.1902
0.8	0.3128	0.0936	0.2192
1.0	0.3129	0.0722	0.2407
1.5	0.3130	0.0388	0.2742
3.0	0.3131	0.0068	0.3063
5.0	0.3131	0.0007	0.3123
	0.3131	0.0	0.3131

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2967	0.2004	0.0963
0.2	0.3022	0.1544	0.1478
0.3	0.3055	0.1243	0.1812
0.4	0.3076	0.1028	0.2048
0.6	0.3101	0.0736	0.2365
0.8	0.3114	0.0548	0.2566
1.0	0.3121	0.0416	0.2704
1.5	0.3128	0.0220	0.2908
3.0	0.3130	0.0038	0.3092
5.0	0.3131	0.0004	0.3126
	0.3131	0.0	0.3131

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2629	0.0663	0.1966
0.2	0.2857	0.0421	0.2436
0.3	0.2958	0.0307	0.2651
0.4	0.3014	0.0239	0.2775
0.6	0.3071	0.0161	0.2910
0.8	0.3097	0.0116	0.2982
1.0	0.3111	0.0086	0.3025
1.5	0.3125	0.0045	0.3080
3.0	0.3130	0.0008	0.3123
5.0	0.3131	0.0001	0.3130
	0.3131	0.0	0.3131

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5926	2.2035	0.3891
0.2	2.6113	1.9131	0.6981
0.3	2.6259	1.6710	0.9549
0.4	2.6374	1.4650	1.1724
0.6	2.6532	1.1353	1.5180
0.8	2.6630	0.8873	1.7757
1.0	2.6690	0.6980	1.9710
1.5	2.6759	0.3909	2.2850
3.0	2.6790	0.0749	2.6041
5.0	2.6792	0.0091	2.6701
	2.6792	0.0	2.6792

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8001	1.2711	0.5291
0.2	2.0157	1.1002	0.9154
0.3	2.1727	0.9587	1.2140
0.4	2.2896	0.8389	1.4507
0.6	2.4449	0.6485	1.7965
0.8	2.5359	0.5060	2.0299
1.0	2.5904	0.3977	2.1928
1.5	2.6513	0.2224	2.4288
3.0	2.6781	0.0426	2.6355
5.0	2.6791	0.0051	2.6740
	2.6792	0.0	2.6792

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.9445	0.2643	0.6802
0.2	1.3767	0.2281	1.1486
0.3	1.6887	0.1982	1.4905
0.4	1.9198	0.1731	1.7467
0.6	2.2246	0.1334	2.0911
0.8	2.4020	0.1040	2.2980
1.0	2.5078	0.0816	2.4262
1.5	2.6254	0.0456	2.5798
3.0	2.6772	0.0087	2.6685
5.0	2.6791	0.0011	2.6781
	2.6792	0.0	2.6792

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4955	1.2711	0.2245
0.2	1.5017	1.1002	0.4015
0.3	1.5066	0.9587	0.5479
0.4	1.5103	0.8389	0.6714
0.6	1.5155	0.6485	0.8671
0.8	1.5187	0.5060	1.0127
1.0	1.5206	0.3977	1.1230
1.5	1.5229	0.2224	1.3004
3.0	1.5239	0.0426	1.4813
5.0	1.5239	0.0051	1.5188
	1.5239	0.0	1.5239

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1927	0.8421	0.3505
0.2	1.2836	0.7006	0.5830
0.3	1.3455	0.5937	0.7518
0.4	1.3895	0.5091	0.8804
0.6	1.4452	0.3833	1.0619
0.8	1.4765	0.2946	1.1819
1.0	1.4948	0.2295	1.2653
1.5	1.5149	0.1271	1.3878
3.0	1.5236	0.0242	1.4994
5.0	1.5239	0.0029	1.5210
	1.5239	0.0	1.5239

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7453	0.2086	0.5367
0.2	0.9908	0.1641	0.8266
0.3	1.1427	0.1341	1.0086
0.4	1.2440	0.1122	1.1319
0.6	1.3653	0.0819	1.2834
0.8	1.4301	0.0619	1.3682
1.0	1.4669	0.0477	1.4192
1.5	1.5064	0.0262	1.4802
3.0	1.5233	0.0050	1.5183
5.0	1.5239	0.0006	1.5233
	1.5239	0.0	1.5239

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3110	0.2643	0.0467
0.2	0.3113	0.2281	0.0832
0.3	0.3115	0.1982	0.1133
0.4	0.3117	0.1731	0.1385
0.6	0.3119	0.1334	0.1784
0.8	0.3120	0.1040	0.2081
1.0	0.3121	0.0816	0.2305
1.5	0.3122	0.0456	0.2666
3.0	0.3122	0.0087	0.3035
5.0	0.3122	0.0011	0.3112
	0.3122	0.0	0.3122

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2954	0.2086	0.0868
0.2	0.3007	0.1641	0.1366
0.3	0.3040	0.1341	0.1698
0.4	0.3062	0.1122	0.1940
0.6	0.3088	0.0819	0.2269
0.8	0.3102	0.0619	0.2483
1.0	0.3110	0.0477	0.2633
1.5	0.3119	0.0262	0.2857
3.0	0.3122	0.0050	0.3073
5.0	0.3122	0.0006	0.3116
	0.3122	0.0	0.3122

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 0.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2572	0.0720	0.1852
0.2	0.2812	0.0466	0.2346
0.3	0.2923	0.0343	0.2580
0.4	0.2985	0.0269	0.2716
0.6	0.3050	0.0183	0.2867
0.8	0.3081	0.0133	0.2948
1.0	0.3098	0.0101	0.2997
1.5	0.3115	0.0054	0.3061
3.0	0.3122	0.0010	0.3112
5.0	0.3122	0.0001	0.3121
	0.3122	0.0	0.3122

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5714	2.2518	0.3197
0.2	2.5776	1.9933	0.5843
0.3	2.5841	1.7730	0.8111
0.4	2.5901	1.5818	1.0083
0.6	2.5997	1.2672	1.3324
0.8	2.6065	1.0219	1.5846
1.0	2.6113	0.8282	1.7830
1.5	2.6176	0.4973	2.1203
3.0	2.6213	0.1147	2.5066
5.0	2.6216	0.0172	2.6043
	2.6216	0.0	2.6216

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7410	1.3034	0.4376
0.2	1.9275	1.1526	0.7749
0.3	2.0704	1.0242	1.0462
0.4	2.1815	0.9128	1.2687
0.6	2.3379	0.7302	1.6077
0.8	2.4364	0.5881	1.8483
1.0	2.4996	0.4763	2.0233
1.5	2.5774	0.2857	2.2917
3.0	2.6192	0.0659	2.5533
5.0	2.6215	0.0099	2.6116
	2.6216	0.0	2.6216

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8379	0.2621	0.5658
0.2	1.2220	0.2404	0.9817
0.3	1.5140	0.2133	1.3007
0.4	1.7400	0.1899	1.5501
0.6	2.0559	0.1517	1.9042
0.8	2.2537	0.1220	2.1317
1.0	2.3799	0.0987	2.2811
1.5	2.5344	0.0592	2.4752
3.0	2.6169	0.0136	2.6033
5.0	2.6215	0.0020	2.6194
	2.6216	0.0	2.6216

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4885	1.3034	0.1850
0.2	1.4905	1.1526	0.3379
0.3	1.4927	1.0242	0.4685
0.4	1.4947	0.9128	0.5816
0.6	1.4979	0.7302	0.7677
0.8	1.5002	0.5882	0.9120
1.0	1.5017	0.4763	1.0254
1.5	1.5038	0.2857	1.2182
3.0	1.5051	0.0659	1.4392
5.0	1.5051	0.0099	1.4952
	1.5051	0.0	1.5051

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1664	0.8733	0.2932
0.2	1.2473	0.7459	0.5014
0.3	1.3056	0.6458	0.6597
0.4	1.3489	0.5644	0.7845
0.6	1.4071	0.4395	0.9676
0.8	1.4422	0.3481	1.0941
1.0	1.4641	0.2790	1.1851
1.5	1.4905	0.1652	1.3253
3.0	1.5044	0.0378	1.4665
5.0	1.5051	0.0057	1.4994
	1.5051	0.0	1.5051

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6773	0.2199	0.4573
0.2	0.9081	0.1786	0.7295
0.3	1.0600	0.1493	0.9106
0.4	1.1660	0.1273	1.0387
0.6	1.2998	0.0959	1.2039
0.8	1.3762	0.0745	1.3017
1.0	1.4222	0.0590	1.3632
1.5	1.4760	0.0344	1.4415
3.0	1.5036	0.0078	1.4958
5.0	1.5051	0.0012	1.5039
	1.5051	0.0	1.5051

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3107	0.2721	0.0386
0.2	0.3108	0.2404	0.0705
0.3	0.3109	0.2133	0.0976
0.4	0.3110	0.1899	0.1211
0.6	0.3111	0.1517	0.1595
0.8	0.3112	0.1220	0.1892
1.0	0.3113	0.0987	0.2126
1.5	0.3114	0.0592	0.2522
3.0	0.3114	0.0136	0.2978
5.0	0.3114	0.0020	0.3094
	0.3114	0.0	0.3114

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2938	0.2200	0.0738
0.2	0.2987	0.1786	0.1201
0.3	0.3019	0.1493	0.1526
0.4	0.3042	0.1273	0.1769
0.6	0.3070	0.0959	0.2111
0.8	0.3087	0.0745	0.2341
1.0	0.3096	0.0590	0.2506
1.5	0.3108	0.0344	0.2764
3.0	0.3114	0.0078	0.3036
5.0	0.3114	0.0012	0.3103
	0.3114	0.0	0.3114

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.2

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2486	0.0807	0.1679
0.2	0.2741	0.0539	0.2202
0.3	0.2865	0.0404	0.2462
0.4	0.2938	0.0321	0.2617
0.6	0.3016	0.0222	0.2793
0.8	0.3055	0.0165	0.2890
1.0	0.3077	0.0128	0.2950
1.5	0.3102	0.0072	0.3029
3.0	0.3114	0.0016	0.3098
5.0	0.3114	0.0002	0.3112
	0.3114	0.0	0.3114

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5703	2.2793	0.2909
0.2	2.5762	2.0404	0.5358
0.3	2.5829	1.8345	0.7484
0.4	2.5891	1.6538	0.9353
0.6	2.5995	1.3519	1.2476
0.8	2.6072	1.1116	1.4956
1.0	2.6128	0.9181	1.6947
1.5	2.6208	0.5771	2.0437
3.0	2.6261	0.1518	2.4743
5.0	2.6265	0.0270	2.5995
	2.6265	0.0	2.6265

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7192	1.3197	0.3995
0.2	1.8949	1.1802	0.7147
0.3	2.0327	1.0599	0.9728
0.4	2.1425	0.9546	1.1879
0.6	2.3016	0.7790	1.5226
0.8	2.4060	0.6397	1.7663
1.0	2.4757	0.5279	1.9478
1.5	2.5667	0.3314	2.2353
3.0	2.6224	0.0871	2.5353
5.0	2.6264	0.0155	2.6109
	2.6265	0.0	2.6265

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7932	0.2755	0.5177
0.2	1.1551	0.2462	0.9089
0.3	1.4367	0.2208	1.2159
0.4	1.6596	0.1986	1.4610
0.6	1.9806	0.1618	1.8188
0.8	2.1899	0.1327	2.0572
1.0	2.3286	0.1094	2.2192
1.5	2.5088	0.0686	2.4402
3.0	2.6184	0.0180	2.6004
5.0	2.6263	0.0032	2.6231
	2.6265	0.0	2.6265

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4881	1.3197	0.1684
0.2	1.4901	1.1802	0.3099
0.3	1.4923	1.0599	0.4324
0.4	1.4944	0.9546	0.5398
0.6	1.4978	0.7790	0.7189
0.8	1.5004	0.6397	0.8607
1.0	1.5023	0.5279	0.9744
1.5	1.5049	0.3314	1.1735
3.0	1.5066	0.0871	1.4195
5.0	1.5068	0.0155	1.4913
	1.5068	0.0	1.5068

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1566	0.8878	0.2688
0.2	1.2335	0.7683	0.4652
0.3	1.2905	0.6729	0.6176
0.4	1.3339	0.5943	0.7396
0.6	1.3939	0.4718	0.9221
0.8	1.4315	0.3806	1.0509
1.0	1.4559	0.3104	1.1454
1.5	1.4869	0.1920	1.2949
3.0	1.5054	0.0500	1.4554
5.0	1.5067	0.0089	1.4978
	1.5068	0.0	1.5068

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6478	0.2250	0.4228
0.2	0.8706	0.1855	0.6850
0.3	1.0215	0.1570	0.8645
0.4	1.1293	0.1352	0.9941
0.6	1.2693	0.1037	1.1656
0.8	1.3521	0.0819	1.2702
1.0	1.4037	0.0659	1.3378
1.5	1.4673	0.0401	1.4272
3.0	1.5041	0.0103	1.4937
5.0	1.5067	0.0018	1.5048
	1.5068	0.0	1.5068

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3107	0.2755	0.0352
0.2	0.3108	0.2462	0.0646
0.3	0.3109	0.2208	0.0901
0.4	0.3110	0.1986	0.1123
0.6	0.3111	0.1618	0.1493
0.8	0.3112	0.1327	0.1785
1.0	0.3113	0.1094	0.2019
1.5	0.3114	0.0686	0.2429
3.0	0.3115	0.0180	0.2935
5.0	0.3115	0.0032	0.3083
	0.3115	0.0	0.3115

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2932	0.2250	0.0681
0.2	0.2979	0.1855	0.1123
0.3	0.3011	0.1570	0.1441
0.4	0.3034	0.1352	0.1682
0.6	0.3064	0.1037	0.2027
0.8	0.3082	0.0819	0.2262
1.0	0.3093	0.0659	0.2433
1.5	0.3107	0.0401	0.2705
3.0	0.3115	0.0103	0.3011
5.0	0.3115	0.0018	0.3097
	0.3115	0.0	0.3115

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 1.8

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2445	0.0849	0.1596
0.2	0.2706	0.0577	0.2130
0.3	0.2837	0.0436	0.2401
0.4	0.2914	0.0349	0.2565
0.6	0.2999	0.0245	0.2754
0.8	0.3043	0.0184	0.2859
1.0	0.3069	0.0144	0.2924
1.5	0.3098	0.0085	0.3013
3.0	0.3114	0.0021	0.3093
5.0	0.3115	0.0004	0.3111
	0.3115	0.0	0.3115

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5667	2.2791	0.2877
0.2	2.5697	2.0397	0.5300
0.3	2.5740	1.8334	0.7406
0.4	2.5782	1.6523	0.9258
0.6	2.5854	1.3498	1.2357
0.8	2.5910	1.1091	1.4819
1.0	2.5951	0.9154	1.6796
1.5	2.6010	0.5744	2.0265
3.0	2.6050	0.1505	2.4544
5.0	2.6053	0.0267	2.5786
	2.6053	0.0	2.6053

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7154	1.3203	0.3951
0.2	1.8882	1.1810	0.7071
0.3	2.0237	1.0608	0.9629
0.4	2.1315	0.9554	1.1761
0.6	2.2877	0.7795	1.5082
0.8	2.3902	0.6400	1.7502
1.0	2.4584	0.5279	1.9305
1.5	2.5472	0.3309	2.2163
3.0	2.6013	0.0867	2.5146
5.0	2.6052	0.0154	2.5898
	2.6053	0.0	2.6053

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7880	0.2758	0.5122
0.2	1.1465	0.2466	0.8999
0.3	1.4257	0.2214	1.2044
0.4	1.6469	0.1922	1.4477
0.6	1.9655	0.1623	1.8032
0.8	2.1732	0.1331	2.0401
1.0	2.3109	0.1097	2.2011
1.5	2.4893	0.0687	2.4206
3.0	2.5973	0.0180	2.5793
5.0	2.6050	0.0032	2.6018
	2.6053	0.0	2.6053

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4869	1.3203	0.1666
0.2	1.4879	1.1810	0.3069
0.3	1.4893	1.0608	0.4285
0.4	1.4907	0.9554	0.5353
0.6	1.4932	0.7795	0.7136
0.8	1.4950	0.6400	0.8551
1.0	1.4964	0.5279	0.9685
1.5	1.4983	0.3309	1.1674
3.0	1.4996	0.0867	1.4130
5.0	1.4997	0.0154	1.4844
	1.4997	0.0	1.4997

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1549	0.8889	0.2660
0.2	1.2307	0.7698	0.4609
0.3	1.2868	0.6746	0.6123
0.4	1.3296	0.5960	0.7336
0.6	1.3888	0.4732	0.9156
0.8	1.4259	0.3818	1.0441
1.0	1.4499	0.3113	1.1386
1.5	1.4803	0.1923	1.2880
3.0	1.4984	0.0499	1.4485
5.0	1.4997	0.0089	1.4909
	1.4997	0.0	1.4997

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6443	0.2256	0.4188
0.2	0.8657	0.1862	0.6795
0.3	1.0159	0.1577	0.8582
0.4	1.1234	0.1359	0.9875
0.6	1.2631	0.1043	1.1588
0.8	1.3457	0.0824	1.2633
1.0	1.3973	0.0664	1.3309
1.5	1.4606	0.0403	1.4203
3.0	1.4971	0.0104	1.4867
5.0	1.4997	0.0018	1.4978
	1.4997	0.0	1.4997

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3107	0.2758	0.0348
0.2	0.3107	0.2466	0.0641
0.3	0.3108	0.2213	0.0894
0.4	0.3108	0.1992	0.1116
0.6	0.3109	0.1623	0.1486
0.8	0.3110	0.1331	0.1779
1.0	0.3111	0.1097	0.2013
1.5	0.3111	0.0687	0.2424
3.0	0.3112	0.0180	0.2932
5.0	0.3112	0.0032	0.3080
	0.3112	0.0	0.3112

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2931	0.2256	0.0675
0.2	0.2977	0.1862	0.1115
0.3	0.3009	0.1577	0.1432
0.4	0.3032	0.1359	0.1673
0.6	0.3061	0.1043	0.2018
0.8	0.3079	0.0824	0.2255
1.0	0.3090	0.0664	0.2427
1.5	0.3104	0.0403	0.2700
3.0	0.3112	0.0104	0.3008
5.0	0.3112	0.0018	0.3094
	0.3112	0.0	0.3112

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2440	0.0854	0.1586
0.2	0.2701	0.0581	0.2120
0.3	0.2832	0.0440	0.2393
0.4	0.2910	0.0352	0.2558
0.6	0.2996	0.0247	0.2748
0.8	0.3040	0.0186	0.2854
1.0	0.3065	0.0146	0.2920
1.5	0.3095	0.0085	0.3009
3.0	0.3111	0.0022	0.3089
5.0	0.3112	0.0004	0.3108
	0.3112	0.0	0.3112

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5659	2.2695	0.2964
0.2	2.5677	2.0230	0.5447
0.3	2.5707	1.8113	0.7594
0.4	2.5739	1.6262	0.9477
0.6	2.5794	1.3186	1.2608
0.8	2.5836	1.0756	1.5079
1.0	2.5867	0.8816	1.7051
1.5	2.5910	0.5439	2.0471
3.0	2.5938	0.1360	2.4578
5.0	2.5940	0.0228	2.5712
	2.5940	0.0	2.5940

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7215	1.3149	0.4066
0.2	1.8970	1.1717	0.7252
0.3	2.0333	1.0486	0.9847
0.4	2.1409	0.9410	1.2000
0.6	2.2950	0.7623	1.5327
0.8	2.3945	0.6214	1.7731
1.0	2.4597	0.5090	1.9507
1.5	2.5427	0.3138	2.2289
3.0	2.5908	0.0785	2.5123
5.0	2.5939	0.0132	2.5808
	2.5940	0.0	2.5940

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8015	0.2748	0.5268
0.2	1.1666	0.2448	0.9219
0.3	1.4488	0.2189	1.2298
0.4	1.6705	0.1963	1.4741
0.6	1.9865	0.1589	1.8276
0.8	2.1896	0.1294	2.0602
1.0	2.3223	0.1060	2.2163
1.5	2.4905	0.0653	2.4252
3.0	2.5875	0.0163	2.5712
5.0	2.5938	0.0027	2.5911
	2.5940	0.0	2.5940

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4866	1.3149	0.1717
0.2	1.4872	1.1717	0.3155
0.3	1.4882	1.0486	0.4396
0.4	1.4893	0.9410	0.5483
0.6	1.4911	0.7623	0.7289
0.8	1.4925	0.6214	0.8711
1.0	1.4936	0.5090	0.9845
1.5	1.4950	0.3138	1.1812
3.0	1.4959	0.0785	1.4175
5.0	1.4960	0.0132	1.4828
	1.4960	0.0	1.4960

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1576	0.8842	0.2734
0.2	1.2344	0.7625	0.4719
0.3	1.2907	0.6656	0.6251
0.4	1.3333	0.5860	0.7473
0.6	1.3915	0.4622	0.9293
0.8	1.4274	0.3704	1.0570
1.0	1.4503	0.3001	1.1502
1.5	1.4788	0.1825	1.2963
3.0	1.4949	0.0453	1.4497
5.0	1.4960	0.0076	1.4884
	1.4960	0.0	1.4960

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6534	0.2240	0.4294
0.2	0.8771	0.1840	0.6931
0.3	1.0276	0.1553	0.8723
0.4	1.1343	0.1333	1.0010
0.6	1.2717	0.1017	1.1700
0.8	1.3520	0.0799	1.2721
1.0	1.4015	0.0640	1.3375
1.5	1.4610	0.0383	1.4227
3.0	1.4938	0.0094	1.4844
5.0	1.4959	0.0016	1.4944
	1.4960	0.0	1.4960

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3106	0.2748	0.0359
0.2	0.3107	0.2448	0.0659
0.3	0.3107	0.2189	0.0918
0.4	0.3108	0.1963	0.1144
0.6	0.3108	0.1589	0.1519
0.8	0.3109	0.1294	0.1815
1.0	0.3109	0.1060	0.2050
1.5	0.3110	0.0653	0.2457
3.0	0.3110	0.0163	0.2947
5.0	0.3110	0.0027	0.3083
	0.3110	0.0	0.3110

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2932	0.2240	0.0693
0.2	0.2979	0.1840	0.1139
0.3	0.3011	0.1553	0.1458
0.4	0.3034	0.1333	0.1700
0.6	0.3063	0.1017	0.2045
0.8	0.3080	0.0799	0.2280
1.0	0.3090	0.0640	0.2451
1.5	0.3103	0.0383	0.2720
3.0	0.3110	0.0094	0.3016
5.0	0.3110	0.0016	0.3095
	0.3110	0.0	0.3110

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 3.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2453	0.0841	0.1612
0.2	0.2713	0.0569	0.2143
0.3	0.2841	0.0429	0.2412
0.4	0.2917	0.0323	0.2574
0.6	0.3000	0.0240	0.2760
0.8	0.3043	0.0180	0.2863
1.0	0.3067	0.0140	0.2927
1.5	0.3095	0.0081	0.3014
3.0	0.3110	0.0020	0.3090
5.0	0.3110	0.0003	0.3107
	0.3110	0.0	0.3110

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5870	2.2913	0.2957
0.2	2.6040	2.0609	0.5431
0.3	2.6187	1.8616	0.7671
0.4	2.6310	1.6859	0.9450
0.6	2.6499	1.3908	1.2591
0.8	2.6630	1.1541	1.5089
1.0	2.6721	0.9622	1.7099
1.5	2.6847	0.6199	2.0648
3.0	2.6933	0.1772	2.5161
5.0	2.6941	0.0357	2.6584
	2.6941	0.0	2.6941

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7293	1.3229	0.4064
0.2	1.9120	1.1866	0.7254
0.3	2.6187	1.8616	0.7671
0.4	2.1700	0.9665	1.2035
0.6	2.3371	0.7948	1.5423
0.8	2.4480	0.6582	1.7899
1.0	2.5229	0.5479	1.9750
1.5	2.6229	0.3523	2.2706
3.0	2.6883	0.1006	2.5877
5.0	2.6939	0.0203	2.6736
	2.6941	0.0	2.6941

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8015	0.2754	0.5261
0.2	1.1678	0.2463	0.9215
0.3	1.4527	0.2214	1.2313
0.4	1.6786	0.1997	1.4789
0.6	2.0058	0.1637	1.8422
0.8	2.2214	0.1352	2.0861
1.0	2.3661	0.1124	2.2537
1.5	2.5582	0.0721	2.4861
3.0	2.6830	0.0206	2.6624
5.0	2.6936	0.0041	2.6895
	2.6941	0.0	2.6941

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4937	1.3230	0.1707
0.2	1.4993	1.1866	0.3127
0.3	1.5042	1.0693	0.4349
0.4	1.5082	0.9665	0.5418
0.6	1.5144	0.7948	0.7196
0.8	1.5187	0.6582	0.8605
1.0	1.5217	0.5479	0.9737
1.5	1.5257	0.3523	1.1734
3.0	1.5285	0.1006	1.4279
5.0	1.5288	0.0203	1.5085
	1.5288	0.0	1.5288

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1612	0.8883	0.2729
0.2	1.2408	0.7700	0.4708
0.3	1.2996	0.6761	0.6235
0.4	1.3445	0.5988	0.7457
0.6	1.4068	0.4784	0.9284
0.8	1.4463	0.3888	1.0574
1.0	1.4721	0.3197	1.1524
1.5	1.5056	0.2022	1.3034
3.0	1.5269	0.0571	1.4698
5.0	1.5287	0.0115	1.5172
	1.5288	0.0	1.5288

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6534	0.2245	0.4289
0.2	0.8778	0.1851	0.6927
0.3	1.0295	0.1569	0.8726
0.4	1.1381	0.1354	1.0027
0.6	1.2796	0.1044	1.1752
0.8	1.3641	0.0830	1.2810
1.0	1.4173	0.0673	1.3499
1.5	1.4840	0.0418	1.4422
3.0	1.5252	0.0117	1.5135
5.0	1.5286	0.0023	1.5263
	1.5288	0.0	1.5288

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3109	0.2754	0.0355
0.2	0.3112	0.2463	0.0649
0.3	0.3114	0.2214	0.0900
0.4	0.3116	0.1997	0.1119
0.6	0.3118	0.1637	0.1482
0.8	0.3120	0.1352	0.1768
1.0	0.3121	0.1124	0.1997
1.5	0.3123	0.0721	0.2402
3.0	0.3124	0.0206	0.2919
5.0	0.3124	0.0041	0.3083
	0.3124	0.0	0.3124

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2935	0.2245	0.0690
0.2	0.2983	0.1851	0.1132
0.3	0.3016	0.1569	0.1447
0.4	0.3039	0.1354	0.1686
0.6	0.3070	0.1044	0.2026
0.8	0.3088	0.0830	0.2258
1.0	0.3100	0.0673	0.2427
1.5	0.3115	0.0418	0.2696
3.0	0.3124	0.0117	0.3007
5.0	0.3124	0.0023	0.3101
	0.3124	0.0	0.3124

Refractive index of scattering medium = 2.00 - 0.60 i

Particle size parameter α = 6.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2453	0.0843	0.1610
0.2	0.2713	0.0572	0.2141
0.3	0.2843	0.0433	0.2410
0.4	0.2920	0.0347	0.2572
0.6	0.3005	0.0245	0.2760
0.8	0.3049	0.0186	0.2864
1.0	0.3075	0.0146	0.2929
1.5	0.3105	0.0088	0.3018
3.0	0.3123	0.0024	0.3099
5.0	0.3124	0.0005	0.3120
	0.3124	0.0	0.3124

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6360	2.1453	0.4907
0.2	2.6789	1.8215	0.8574
0.3	2.7089	1.5590	1.1499
0.4	2.7303	1.3510	1.3893
0.6	2.7574	1.0033	1.7541
0.8	2.7722	0.7593	2.0129
1.0	2.7806	0.5797	2.2008
1.5	2.7890	0.3034	2.4856
3.0	2.7922	0.0490	2.7431
5.0	2.7923	0.0049	2.7873
	2.7923	0.0	2.7923

Isotropic Case

Scattering to Extinction Ratio = 0.1

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8901	1.2288	0.6613
0.2	2.1449	1.0361	1.1088
0.3	2.3196	0.8825	1.4370
0.4	2.4433	0.7566	1.6867
0.6	2.5974	0.5636	2.0338
0.8	2.6808	0.4255	2.2553
1.0	2.7273	0.3245	2.4029
1.5	2.7745	0.1696	2.6049
3.0	2.7918	0.0274	2.7644
5.0	2.7923	0.0028	2.7895
	2.7923	0.0	2.7923

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.0965	0.2536	0.8428
0.2	1.5848	0.2123	1.3725
0.3	1.9152	0.1799	1.7353
0.4	2.1472	0.1537	1.9935
0.6	2.4338	0.1140	2.3199
0.8	2.5878	0.0859	2.5019
1.0	2.6733	0.0654	2.6079
1.5	2.7597	0.0341	2.7256
3.0	2.7914	0.0055	2.7859
5.0	2.7923	0.0006	2.7917
	2.7923	0.0	2.7923

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5099	1.2288	0.2811
0.2	1.5239	1.0361	0.4877
0.3	1.5335	0.8825	0.6510
0.4	1.5404	0.7566	0.7838
0.6	1.5489	0.5636	0.9854
0.8	1.5536	0.4255	1.1281
1.0	1.5562	0.3245	1.2318
1.5	1.5589	0.1696	1.3893
3.0	1.5599	0.2740	1.5325
5.0	1.5599	0.0028	1.5571
	1.5599	0.0	1.5599

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2315	0.8006	0.4309
0.2	1.3348	0.6448	0.6900
0.3	1.4005	0.5328	0.8676
0.4	1.4446	0.4473	0.9973
0.6	1.4971	0.3249	1.1723
0.8	1.5245	0.2420	1.2825
1.0	1.5394	0.1831	1.3563
1.5	1.5543	0.0950	1.4593
3.0	1.5597	0.0153	1.5444
5.0	1.5599	0.0015	1.5583
	1.5599	0.0	1.5599

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8368	0.1936	0.6433
0.2	1.0942	0.1466	0.9476
0.3	1.2421	0.1167	1.1254
0.4	1.3357	0.0956	1.2401
0.6	1.4413	0.0675	1.3738
0.8	1.4939	0.0496	1.4444
1.0	1.5221	0.0372	1.4848
1.5	1.5497	0.0192	1.5305
3.0	1.5596	0.0031	1.5565
5.0	1.5599	0.0003	1.5596
	1.5599	0.0	1.5599

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3116	0.2536	0.0580
0.2	0.3122	0.2123	0.0999
0.3	0.3126	0.1799	0.1327
0.4	0.3129	0.1537	0.1592
0.6	0.3133	0.1140	0.1993
0.8	0.3135	0.0859	0.2276
1.0	0.3136	0.0654	0.2482
1.5	0.3137	0.0341	0.2796
3.0	0.3137	0.0055	0.3082
5.0	0.3137	0.0006	0.3132
	0.3137	0.0	0.3137

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2978	0.1936	0.1042
0.2	0.3034	0.1466	0.1569
0.3	0.3067	0.1167	0.1900
0.4	0.3088	0.0956	0.2132
0.6	0.3111	0.0675	0.2436
0.8	0.3123	0.0496	0.2627
1.0	0.3129	0.0372	0.2757
1.5	0.3135	0.0192	0.2943
3.0	0.3137	0.0031	0.3106
5.0	0.3137	0.0003	0.3134
	0.3137	0.0	0.3137

Isotropic Case

Scattering to Extinction Ratio = 0.065

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2673	0.0618	0.2054
0.2	0.2890	0.0387	0.2503
0.3	0.2984	0.0280	0.2703
0.4	0.3035	0.0217	0.2818
0.6	0.3086	0.0145	0.2942
0.8	0.3110	0.0103	0.3006
1.0	0.3122	0.0076	0.3045
1.5	0.3133	0.0039	0.3094
3.0	0.3137	0.0006	0.3131
5.0	0.3137	0.0001	0.3137
	0.3137	0.0	0.3137

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6125	2.1686	0.4440
0.2	2.6422	1.8570	0.7852
0.3	2.6638	1.6013	1.0625
0.4	2.6798	1.3869	1.2929
0.6	2.7006	1.0504	1.6502
0.8	2.7125	0.8036	1.9089
1.0	2.7194	0.6195	2.0999
1.5	2.7268	0.3312	2.3956
3.0	2.7297	0.0562	2.6735
5.0	2.7298	0.0059	2.7239
	2.7283	0.0	2.7283

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8466	1.2469	0.5996
0.2	2.0826	1.0627	1.0200
0.3	2.2488	0.9132	1.3356
0.4	2.3689	0.7889	1.5801
0.6	2.5226	0.5955	1.9271
0.8	2.6084	0.4547	2.1537
1.0	2.6576	0.3502	2.3074
1.5	2.7091	0.1870	2.5221
3.0	2.7292	0.0317	2.6975
5.0	2.7298	0.0033	2.7265
	2.7283	0.0	2.7283

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.0250	0.2584	0.7666
0.2	1.4883	0.2191	1.2692
0.3	1.8111	0.1876	1.6235
0.4	2.0429	0.1616	1.8813
0.6	2.3372	0.1216	2.2156
0.8	2.5004	0.0927	2.4078
1.0	2.5935	0.0713	2.5222
1.5	2.6908	0.0380	2.6528
3.0	2.7287	0.0064	2.7222
5.0	2.7298	0.0007	2.7291
	2.7283	0.0	2.7283

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5022	1.2469	0.2553
0.2	1.5119	1.0626	0.4493
0.3	1.5190	0.9131	0.6059
0.4	1.5241	0.7888	0.7353
0.6	1.5309	0.5954	0.9354
0.8	1.5347	0.4546	1.0800
1.0	1.5369	0.3501	1.1868
1.5	1.5392	0.1869	1.3523
3.0	1.5402	0.0317	1.5085
5.0	1.5402	0.0033	1.5369
	1.5397	0.0	1.5397

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2129	0.8190	0.3938
0.2	1.3104	0.6687	0.6418
0.3	1.3743	0.5581	0.8162
0.4	1.4183	0.4723	0.9460
0.6	1.4720	0.3475	1.1245
0.8	1.5008	0.2616	1.2392
1.0	1.5169	0.1999	1.3170
1.5	1.5336	0.1058	1.4277
3.0	1.5400	0.0179	1.5221
5.0	1.5402	0.0019	1.5383
	1.5397	0.0	1.5397

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7946	0.2003	0.5942
0.2	1.0473	0.1542	0.8931
0.3	1.1975	0.1240	1.0734
0.4	1.2946	0.1024	1.1922
0.6	1.4069	0.0732	1.3337
0.8	1.4644	0.0543	1.4101
1.0	1.4958	0.0411	1.4547
1.5	1.5277	0.0216	1.5061
3.0	1.5398	0.0036	1.5362
5.0	1.5402	0.0004	1.5398
	1.5397	0.0	1.5397

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3113	0.2584	0.0529
0.2	0.3117	0.2191	0.0926
0.3	0.3120	0.1876	0.1245
0.4	0.3122	0.1616	0.1506
0.6	0.3125	0.1216	0.1910
0.8	0.3127	0.0926	0.2201
1.0	0.3128	0.0713	0.2415
1.5	0.3129	0.0380	0.2749
3.0	0.3129	0.0064	0.3065
5.0	0.3129	0.0007	0.3122
	0.3129	0.0	0.3129

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2967	0.2003	0.0963
0.2	0.3022	0.1542	0.1480
0.3	0.3054	0.1240	0.1814
0.4	0.3075	0.1024	0.2051
0.6	0.3100	0.0732	0.2368
0.8	0.3113	0.0543	0.2570
1.0	0.3119	0.0411	0.2708
1.5	0.3126	0.0216	0.2911
3.0	0.3129	0.0036	0.3093
5.0	0.3129	0.0004	0.3125
	0.3129	0.0	0.3129

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2628	0.0663	0.1965
0.2	0.2856	0.0420	0.2436
0.3	0.2957	0.0306	0.2651
0.4	0.3013	0.0238	0.2775
0.6	0.3070	0.0160	0.2910
0.8	0.3097	0.0115	0.2982
1.0	0.3110	0.0085	0.3025
1.5	0.3124	0.0044	0.3080
3.0	0.3129	0.0007	0.3122
5.0	0.3129	0.0001	0.3128
	0.3129	0.0	0.3129

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5855	2.1951	0.3904
0.2	2.5989	1.8987	0.7002
0.3	2.6095	1.6520	0.9575
0.4	2.6178	1.4427	1.1751
0.6	2.6294	1.1094	1.5201
0.8	2.6365	0.8602	1.7763
1.0	2.6409	0.6715	1.9694
1.5	2.6459	0.3688	2.2771
3.0	2.6481	0.0679	2.5813
5.0	2.6482	0.0075	2.6406
	2.6481	0.0	2.6481

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7981	1.2677	0.5304
0.2	2.0111	1.0941	0.9169
0.3	2.1653	0.9503	1.2150
0.4	2.2795	0.8288	1.4507
0.6	2.4298	0.6361	1.7937
0.8	2.5166	0.4927	2.0224
1.0	2.5679	0.3843	2.1836
1.5	2.6239	0.2109	2.4130
3.0	2.6474	0.0382	2.6092
5.0	2.6481	0.0043	2.6438
	2.6481	0.0	2.6481

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.9459	0.2640	0.6819
0.2	1.3778	0.2273	1.1505
0.3	1.6884	0.1971	1.4913
0.4	1.9173	0.1716	1.7457
0.6	2.2169	0.1314	2.0854
0.8	2.3890	0.1017	2.2873
1.0	2.4903	0.0793	2.4111
1.5	2.6005	0.0435	2.5571
3.0	2.6466	0.0079	2.6387
5.0	2.6481	0.0009	2.6472
	2.6481	0.0	2.6481

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4932	1.2677	0.2254
0.2	1.4976	1.0941	0.4035
0.3	1.5012	0.9503	0.5508
0.4	1.5039	0.8288	0.6751
0.6	1.5077	0.6361	0.8716
0.8	1.5101	0.4927	1.0174
1.0	1.5115	0.3843	1.1272
1.5	1.5131	0.2109	1.3022
3.0	1.5138	0.0382	1.4757
5.0	1.5139	0.0043	1.5096
	1.5139	0.0	1.5139

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1918	0.8402	0.3515
0.2	1.2817	0.6973	0.5844
0.3	1.3427	0.5893	0.7534
0.4	1.3857	0.5039	0.8819
0.6	1.4399	0.3770	1.0629
0.8	1.4699	0.2878	1.1822
1.0	1.4873	0.2226	1.2647
1.5	1.5059	0.1210	1.3848
3.0	1.5136	0.0218	1.4918
5.0	1.5139	0.0025	1.5114
	1.5139	0.0	1.5139

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7462	0.2082	0.5380
0.2	0.9913	0.1635	0.8278
0.3	1.1426	0.1334	1.0092
0.4	1.2430	0.1113	1.1317
0.6	1.3623	0.0808	1.2816
0.8	1.4255	0.0607	1.3648
1.0	1.4609	0.0465	1.4144
1.5	1.4982	0.0250	1.4731
3.0	1.5133	0.0045	1.5089
5.0	1.5139	0.0005	1.5133
	1.5139	0.0	1.5139

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3109	0.2640	0.0469
0.2	0.3111	0.2273	0.0838
0.3	0.3113	0.1971	0.1142
0.4	0.3114	0.1716	0.1398
0.6	0.3116	0.1314	0.1801
0.8	0.3117	0.1017	0.2100
1.0	0.3117	0.0793	0.2325
1.5	0.3118	0.0435	0.2683
3.0	0.3118	0.0079	0.3039
5.0	0.3118	0.0009	0.3109
	0.3118	0.0	0.3118

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2954	0.2082	0.0871
0.2	0.3006	0.1635	0.1371
0.3	0.3038	0.1334	0.1705
0.4	0.3060	0.1113	0.1947
0.6	0.3085	0.0808	0.2278
0.8	0.3099	0.0607	0.2492
1.0	0.3107	0.0465	0.2642
1.5	0.3115	0.0250	0.2864
3.0	0.3118	0.0045	0.3073
5.0	0.3118	0.0005	0.3113
	0.3118	0.0	0.3118

Isotropic Case

Scattering to Extinction Ratio = 0.285

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2573	0.0718	0.1855
0.2	0.2813	0.0464	0.2349
0.3	0.2923	0.0341	0.2581
0.4	0.2984	0.0267	0.2717
0.6	0.3048	0.1807	0.2868
0.8	0.3079	0.1310	0.2948
1.0	0.3095	0.0099	0.2997
1.5	0.3111	0.0052	0.3059
3.0	0.3118	0.0009	0.3109
5.0	0.3118	0.0001	0.3117
	0.3118	0.0	0.3118

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5386	2.2416	0.2969
0.2	2.5206	1.9745	0.5461
0.3	2.5087	1.7472	0.7615
0.4	2.5004	1.5505	0.9499
0.6	2.4900	1.2284	1.2616
0.8	2.4843	0.9792	1.5051
1.0	2.4812	0.7843	1.6969
1.5	2.4779	0.4567	2.0212
3.0	2.4765	0.0959	2.3806
5.0	2.4765	0.0127	2.4638
	2.4765	0.0	2.4765

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7111	1.3046	0.4065
0.2	1.8767	1.1526	0.7241
0.3	2.0035	1.0219	0.9816
0.4	2.1020	0.9081	1.1939
0.6	2.2395	0.7207	1.5188
0.8	2.3250	0.5750	1.7500
1.0	2.3789	0.4608	1.9181
1.5	2.4432	0.2685	2.1747
3.0	2.4750	0.0564	2.4186
5.0	2.4765	0.0075	2.4690
	2.4765	0.0	2.4765

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8010	0.2739	0.5271
0.2	1.1640	0.2428	0.9212
0.3	1.4421	0.2158	1.2263
0.4	1.6580	0.1920	1.4659
0.6	1.9593	0.1527	1.8066
0.8	2.1464	0.1220	2.0245
1.0	2.2642	0.0978	2.1664
1.5	2.4042	0.0570	2.3472
3.0	2.4733	0.0120	2.4613
5.0	2.4764	0.0016	2.4748
	2.4765	0.0	2.4765

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4774	1.3046	0.1728
0.2	1.4713	1.1526	0.3187
0.3	1.4672	1.0219	0.4454
0.4	1.4644	0.9081	0.5563
0.6	1.4608	0.7207	0.7401
0.8	1.4589	0.5750	0.88[illegible]8
1.0	1.4578	0.4608	0.9970
1.5	1.4566	0.2685	1.1882
3.0	1.4562	0.0564	1.3998
5.0	1.4562	0.0075	1.4487
	1.4562	0.0	1.4562

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1529	0.8790	0.2739
0.2	1.2258	0.7528	0.4730
0.3	1.2787	0.6522	0.6265
0.4	1.3181	0.5694	0.7487
0.6	1.3709	0.4411	0.9297
0.8	1.4024	0.3469	1.0556
1.0	1.4219	0.2754	1.1464
1.5	1.4446	0.1587	1.2858
3.0	1.4556	0.0332	1.4225
5.0	1.4561	0.0044	1.4517
	1.4562	0.0	1.4562

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6530	0.2233	0.4297
0.2	0.8756	0.1827	0.6930
0.3	1.0242	0.1532	0.8709
0.4	1.1286	0.1307	0.9978
0.6	1.2605	0.0982	1.1623
0.8	1.3354	0.0759	1.2595
1.0	1.3801	0.0596	1.3205
1.5	1.4309	0.0339	1.3969
3.0	1.4550	0.0070	1.4480
5.0	1.4561	0.0009	1.4552
	1.4562	0.0	1.4562

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3102	0.2739	0.0363
0.2	0.3100	0.2428	0.0672
0.3	0.3098	0.2158	0.0940
0.4	0.3097	0.1920	0.1176
0.6	0.3095	0.1527	0.1568
0.8	0.3094	0.1220	0.1875
1.0	0.3094	0.0978	0.2116
1.5	0.3093	0.0570	0.2523
3.0	0.3093	0.0120	0.2973
5.0	0.3093	0.0016	0.3077
	0.3093	0.0	0.3093

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2929	0.2233	0.0696
0.2	0.2974	0.1827	0.1148
0.3	0.3004	0.1532	0.1472
0.4	0.3026	0.1307	0.1719
0.6	0.3053	0.0982	0.2070
0.8	0.3068	0.0759	0.2309
1.0	0.3077	0.0596	0.2481
1.5	0.3088	0.0339	0.2748
3.0	0.3093	0.0070	0.3022
5.0	0.3093	0.0009	0.3084
	0.3093	0.0	0.3093

Isotropic Case

Scattering to Extinction Ratio = 0.475

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2452	0.0839	0.1614
0.2	0.2711	0.0566	0.2146
0.3	0.2839	0.0425	0.2414
0.4	0.2913	0.0337	0.2576
0.6	0.2994	0.0233	0.2761
0.8	0.3035	0.0172	0.2862
1.0	0.3057	0.0132	0.2925
1.5	0.3081	0.0073	0.3008
3.0	0.3092	0.0015	0.3077
5.0	0.3093	0.0002	0.3091
	0.3093	0.0	0.3093

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5058	2.2740	0.2317
0.2	2.4636	0.0298	0.4338
0.3	2.4329	1.8191	0.6138
0.4	2.4097	1.6344	0.7753
0.6	2.3778	1.3261	1.0516
0.8	2.3581	1.0815	1.2766
1.0	2.3456	0.8853	1.4604
1.5	2.3305	0.5426	1.7879
3.0	2.3225	0.1307	2.1918
5.0	2.3220	0.0204	2.3016
	2.3220	0.0	2.3220

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.6496	1.3306	0.3190
0.2	1.7767	1.1961	0.5806
0.3	1.8784	1.0774	0.8009
0.4	1.9603	0.9718	0.9885
0.6	2.0803	0.7927	1.2876
0.8	2.1596	0.6487	1.5109
1.0	2.2124	0.5321	1.6803
1.5	2.2803	0.3270	1.9534
3.0	2.3195	0.0790	2.2407
5.0	2.3219	0.0123	2.3096
	2.3220	0.0	2.3220

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6972	0.2811	0.4161
0.2	1.0011	0.2547	0.7464
0.3	1.2453	0.2307	1.0146
0.4	1.4430	0.2090	1.2339
0.6	1.7339	0.1716	1.5623
0.8	1.9268	0.1410	1.7858
1.0	2.0553	0.1159	1.9394
1.5	2.2208	0.0714	2.1494
3.00	2.3161	0.0173	2.2988
5.0	2.3218	0.0027	2.3191
	2.3220	0.0	2.3220

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4662	1.3306	0.1356
0.2	1.4517	1.1961	0.2556
0.3	1.4410	1.0774	0.3635
0.4	1.4328	0.9718	0.4610
0.6	1.4215	0.7928	0.6287
0.8	1.4144	0.6487	0.7657
1.0	1.4099	0.5321	0.8778
1.5	1.4044	0.3270	1.0774
3.0	1.4015	0.0789	1.3226
5.0	1.4013	0.0123	1.3890
	1.4013	0.0	1.4013

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1247	0.9072	0.2175
0.2	1.1823	0.7959	0.3864
0.3	1.2265	0.7035	0.5230
0.4	1.2609	0.6251	0.6358
0.6	1.3095	0.4990	0.8105
0.8	1.3405	0.4026	0.9379
1.0	1.3606	0.3272	1.0334
1.5	1.3861	0.1988	1.1873
3.0	1.4004	0.0476	1.3528
5.0	1.4013	0.0074	1.3939
	1.4013	0.0	1.4013

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.5823	0.2348	0.3475
0.2	0.7801	0.1984	0.5817
0.3	0.9208	0.1706	0.7502
0.4	1.0246	0.1484	0.8762
0.6	1.1632	0.1151	1.0481
0.8	1.2470	0.0912	1.1557
1.0	1.2996	0.0733	1.2263
1.5	1.3638	0.0439	1.3200
3.0	1.3992	0.0104	1.3887
5.0	1.4013	0.0016	1.3996
	1.4013	0.0	1.4013

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3097	0.2811	0.0286
0.2	0.3091	0.2547	0.0544
0.3	0.3086	0.2307	0.0779
0.4	0.3082	0.2091	0.0992
0.6	0.3077	0.1716	0.1361
0.8	0.3074	0.1410	0.1664
1.0	0.3072	0.1159	0.1912
1.5	0.3069	0.0715	0.2354
3.0	0.3067	0.0173	0.2895
5.0	0.3067	0.0027	0.3040
	0.3067	0.0	0.3067

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2911	0.2348	0.0563
0.2	0.2948	0.1984	0.0963
0.3	0.2975	0.1706	0.1268
0.4	0.2994	0.1484	0.1510
0.6	0.3021	0.1151	0.1870
0.8	0.3037	0.0912	0.2125
1.0	0.3047	0.0733	0.2315
1.5	0.3060	0.0439	0.2621
3.0	0.3070	0.0104	0.2963
5.0	0.3067	0.0016	0.3051
	0.3067	0.0	0.3067

Isotropic Case

Scattering to Extinction Ratio = 0.6

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2345	0.0946	0.1400
0.2	0.2612	0.0664	0.1948
0.3	0.2753	0.0510	0.2243
0.4	0.2839	0.0411	0.2428
0.6	0.2936	0.0291	0.2645
0.8	0.2986	0.0218	0.2768
1.0	0.3016	0.0170	0.2846
1.5	0.3049	0.0098	0.2951
3.0	0.3066	0.0023	0.3044
5.0	0.3067	0.0004	0.3064
	0.3067	0.0	0.3067

Particle size parameter $\alpha = 0$

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6502	2.1313	0.5189
0.2	2.7007	1.8005	0.9002
0.3	2.7353	1.5343	1.2010
0.4	2.7597	1.3146	1.4451
0.6	2.7900	0.9767	1.8133
0.8	2.8062	0.7347	2.0716
1.0	2.8152	0.5579	2.2573
1.5	2.8242	0.2887	2.5355
3.0	2.8273	0.0455	2.7819
5.0	2.8274	0.0046	2.8230

Particle Size Parameter $\alpha = 0$

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.9157	1.2180	0.6977
0.2	2.1808	1.0206	1.1602
0.3	2.3598	0.8649	1.4949
0.4	2.4851	0.7382	1.7469
0.6	2.6392	0.5458	2.0933
0.8	2.7211	0.4095	2.3116
1.0	2.7662	0.3105	2.4556
1.5	2.8111	0.1605	2.6506
3.0	2.8270	0.0253	2.8018
5.0	2.8274	0.0025	2.8250

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.1

Reflectivity Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.1379	0.2508	0.8871
0.2	1.6393	0.2084	1.4309
0.3	1.9731	0.1756	1.7976
0.4	2.2046	0.1492	2.0553
0.6	2.4865	0.1098	2.3767
0.8	2.6354	0.0823	2.5532
1.0	2.7170	0.0622	2.6548
1.5	2.7979	0.0321	2.7658
3.0	2.8267	0.0051	2.8217
5.0	2.8274	0.0005	2.8269

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.5

Reflectivity Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5145	1.2180	0.2965
0.2	1.5309	1.0206	0.5103
0.3	1.5419	0.8649	0.6770
0.4	1.5497	0.7382	0.8115
0.6	1.5592	0.5458	1.0134
0.8	1.5642	0.4095	1.1547
1.0	1.5670	0.3105	1.2565
1.5	1.5698	0.1605	1.4093
3.0	1.5708	0.0253	1.5455
5.0	1.5708	0.0025	1.5683

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.5

Reflectivity Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2423	0.7899	0.4525
0.2	1.3486	0.6312	0.7175
0.3	1.4150	0.5186	0.8964
0.4	1.4591	0.4334	1.0257
0.6	1.5109	0.3125	1.1984
0.8	1.5374	0.2314	1.3061
1.0	1.5517	0.1742	1.3775
1.5	1.5657	0.0894	1.4764
3.0	1.5707	0.0140	1.5566
5.0	1.5708	0.0014	1.5694

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.5

Reflectivity Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8607	0.1897	0.6710
0.2	1.1199	0.1423	0.9775
0.3	1.2662	0.1127	1.1536
0.4	1.3577	0.0919	1.2658
0.6	1.4596	0.0645	1.3952
0.8	1.5097	0.0471	1.4626
1.0	1.5361	0.0352	1.5009
1.5	1.5617	0.0179	1.5437
3.0	1.5706	0.0028	1.5678
5.0	1.5708	0.0003	1.5705

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.9

Reflectivity Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3118	0.2508	0.0611
0.2	0.3125	0.2084	0.1042
0.3	0.3130	0.1756	0.1374
0.4	0.3133	0.1492	0.1641
0.6	0.3137	0.1098	0.2039
0.8	0.3139	0.0822	0.2317
1.0	0.3140	0.0622	0.2518
1.5	0.3141	0.0321	0.2820
3.0	0.3142	0.0051	0.3091
5.0	0.3142	0.0005	0.3137

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.9

Reflectivity Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2984	0.1897	0.1087
0.2	0.3041	0.1423	0.1618
0.3	0.3074	0.1127	0.1947
0.4	0.3094	0.0919	0.2175
0.6	0.3117	0.0645	0.2472
0.8	0.3128	0.0471	0.2657
1.0	0.3134	0.0352	0.2782
1.5	0.3140	0.0179	0.2960
3.0	0.3142	0.0028	0.3113
5.0	0.3142	0.0003	0.3139

Particle size parameter $\alpha = 0$

Reflectivity Wall 1 = 0.9

Reflectivity Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2697	0.0594	0.2102
0.2	0.2907	0.0370	0.2538
0.3	0.2997	0.0267	0.2731
0.4	0.3046	0.0206	0.2840
0.6	0.3094	0.0137	0.2958
0.8	0.3116	0.0097	0.3019
1.0	0.3127	0.0072	0.3056
1.5	0.3138	0.0036	0.3101
3.0	0.3142	0.0006	0.3136
5.0	0.3142	0.0001	0.3141

APPENDIX C

TABULATED RESULTS FOR RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5730	2.2697	0.3032
0.6	2.6127	1.3334	1.2793
1.0	2.6293	0.8984	1.7309
3.0	2.6452	0.1431	2.5021
5.0	2.6457	0.0250	2.6207

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7297	1.3130	0.4166
0.6	2.3238	0.7659	1.5579
1.0	2.4986	0.5147	1.9839
3.0	2.6419	0.0817	2.5601
5.0	2.6456	0.0142	2.6313

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8137	0.2739	0.5397
0.6	2.0146	0.1585	1.8561
1.0	2.3595	0.1062	2.2532
3.0	2.6384	0.0168	2.6216
5.0	2.6455	0.0029	2.6426

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4889	1.3135	0.1754
0.6	1.5022	0.7666	0.7355
1.0	1.5077	0.5151	0.9925
3.0	1.5129	0.0818	1.4310
5.0	1.5130	0.0143	1.4987

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1613	0.8816	0.2797
0.6	1.4020	0.4621	0.9399
1.0	1.4637	0.3015	1.1622
3.0	1.5118	0.0468	1.4650
5.0	1.5130	0.0081	1.5048

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6614	0.2226	0.4387
0.6	1.2832	0.1010	1.1822
1.0	1.4149	0.0637	1.3512
3.0	1.5106	0.0096	1.5010
5.0	1.5130	0.0016	1.5113

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3107	0.2741	0.0366
0.6	0.3113	0.1588	0.1524
1.0	0.3115	0.1064	0.2050
3.0	0.3117	0.0168	0.2949
5.0	0.3117	0.0029	0.3088

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2934	0.2227	0.0706
0.6	0.3067	0.1011	0.2056
1.0	0.3096	0.0637	0.2458
3.0	0.3117	0.0096	0.3020
5.0	0.3117	0.0016	0.3100

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2464	0.0829	0.1634
0.6	0.3006	0.0236	0.2770
1.0	0.3073	0.0138	0.2935
3.0	0.3116	0.0019	0.3096
5.0	0.3117	0.0003	0.3114

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 2.4

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3107	0.2741	0.0366
0.6	0.3113	0.1588	0.1524
1.0	0.3115	0.1064	0.2050
3.0	0.3117	0.0168	0.2949
5.0	0.3117	0.0029	0.3088

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2934	0.2227	0.0706
0.6	0.3067	0.1011	0.2056
1.0	0.3096	0.0637	0.2458
3.0	0.3117	0.0096	0.3020
5.0	0.3117	0.0016	0.3100

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2464	0.0829	0.1634
0.6	0.3006	0.0236	0.2770
1.0	0.3073	0.0138	0.2935
3.0	0.3116	0.0019	0.3096
5.0	0.3117	0.0003	0.3114

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.5940	2.3135	0.2805
0.6	2.6883	1.4829	1.2053
1.0	2.7240	1.0698	1.6541
3.0	2.7619	0.2386	2.5232
5.0	2.7638	0.0589	2.7048

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.7202	1.3345	0.3857
0.6	2.3344	0.8428	1.4915
1.0	2.5408	0.6046	1.9361
3.0	2.7528	0.1341	2.6187
5.0	2.7632	0.0331	2.7301

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.7769	0.2775	0.4993
0.6	1.9637	0.1725	1.7912
1.0	2.3511	0.1230	2.2280
3.0	2.7435	0.0271	2.7164
5.0	2.7627	0.0066	2.7560

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.4960	1.3342	0.1617
0.6	1.5269	0.8422	0.6846
1.0	1.5383	0.6041	0.9341
3.0	1.5503	0.1339	1.4163
5.0	1.5509	0.0330	1.5178

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.1570	0.8976	0.2594
0.6	1.4058	0.5075	0.8982
1.0	1.4781	0.3517	1.1263
3.0	1.5475	0.0753	1.4721
5.0	1.5507	0.0185	1.5322

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.6369	0.2275	0.4094
0.6	1.2623	0.1109	1.1514
1.0	1.4118	0.0739	1.3379
3.0	1.5445	0.0152	1.5292
5.0	1.5506	0.0037	1.5468

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3110	0.2774	0.0336
0.6	0.3123	0.1723	0.1400
1.0	0.3128	0.1228	0.1899
3.0	0.3133	0.0270	0.2862
5.0	0.3133	0.0066	0.3066

Reflectivity of Wall 2 = 0.5

0.1	0.2931	0.2274	0.0657
0.6	0.3069	0.1108	0.1961
1.0	0.3102	0.0738	0.2364
3.0	0.3132	0.0152	0.2979
5.0	0.3133	0.0037	0.3095

Reflectivity of Wall 2 = 0.9

0.1	0.2429	0.0867	0.1561
0.6	0.2995	0.0263	0.2732
1.0	0.3072	0.0160	0.2911
3.0	0.3130	0.0030	0.3099
5.0	0.3133	0.0007	0.3125

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.1

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	2.6194	2.1642	0.4552
0.6	2.7192	1.0522	1.6669
1.0	2.7403	0.6198	2.1205
3.0	2.7516	0.0557	2.6959
5.0	2.7517	0.0059	2.7458

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.8574	1.2443	0.6130
0.6	2.5415	0.5965	1.9449
1.0	2.6788	0.3503	2.3285
3.0	2.7511	0.0314	2.7197
5.0	2.7517	0.0033	2.7484

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	1.0402	0.2578	0.7823
0.6	2.3564	0.1218	2.2346
1.0	2.6151	0.0713	2.5438
3.0	2.7506	0.0063	2.7442
5.0	2.7517	0.0006	2.7510

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.5

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	1.5044	1.2429	0.2614
0.6	1.5368	0.5947	0.9421
1.0	1.5435	0.3491	1.1944
3.0	1.5471	0.0313	1.5157
5.0	1.5471	0.0033	1.5438

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	1.2175	0.8156	0.4018
0.6	1.4784	0.3470	1.1313
1.0	1.5238	0.1993	1.3245
3.0	1.5469	0.0176	1.5292
5.0	1.5471	0.0018	1.5452

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.8036	0.1992	0.6044
0.6	1.4137	0.0730	1.3407
1.0	1.5030	0.0409	1.4620
3.0	1.5468	0.0035	1.5432
5.0	1.5471	0.0003	1.5467

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS
Third Order Approximation

Isotropic Case

Scattering to Extinction Ratio = 0.17

Reflectivity of Wall 1 = 0.9

Reflectivity of Wall 2 = 0.1

Optical Spacing	M	N	Q
0.1	0.3114	0.2572	0.0541
0.6	0.3127	0.1210	0.1917
1.0	0.3130	0.0708	0.2422
3.0	0.3132	0.0063	0.3068
5.0	0.3132	0.0006	0.3125

Reflectivity of Wall 2 = 0.5

Optical Spacing	M	N	Q
0.1	0.2969	0.1989	0.0980
0.6	0.3102	0.0728	0.2374
1.0	0.3122	0.0408	0.2713
3.0	0.3131	0.0035	0.3096
5.0	0.3132	0.0003	0.3128

Reflectivity of Wall 2 = 0.9

Optical Spacing	M	N	Q
0.1	0.2637	0.0654	0.1983
0.6	0.3073	0.0158	0.2914
1.0	0.3113	0.0084	0.3028
3.0	0.3131	0.0007	0.3124
5.0	0.3132	0.0000	0.3131

APPENDIX D

Results for the Case of Radiative Equilibrium

RADIANT HEAT TRANSFER BETWEEN PARALLEL WALLS

Value of Parameter M^e

Case of Radiative Equilibrium

Reflectivity of Wall 1 = 0.1

Optical Spacing	Reflectivity Wall 2		
	0.1	0.5	0.9
0.1	2.3894	1.4256	0.3079
0.2	2.2442	1.3726	0.3054
0.3	2.1199	1.3251	0.3030
0.4	2.0109	1.2817	0.3007
0.6	1.8266	1.2043	0.2963
0.8	1.6757	1.1368	0.2921
1.0	1.5491	1.0772	0.2881
1.5	1.3053	0.9534	0.2786
3.0	0.8894	0.7108	0.2538
5.0	0.6249	0.5313	0.2271

Value of Parameter M^e

Case of Radiative Equilibrium

Reflectivity Wall 1 = 0.5

Optical Spacing	Reflectivity Wall 2		
	0.1	0.5	0.9
0.1	1.4256	1.0159	0.2832
0.2	1.3726	0.9887	0.2811
0.3	1.3251	0.9638	0.2791
0.4	1.2817	0.9406	0.2772
0.6	1.2042	0.8982	0.2734
0.8	1.1367	0.8602	0.2698
1.0	1.0770	0.8256	0.2664
1.5	0.9532	0.7508	0.2582
3.0	0.7106	0.5918	0.2368
5.0	0.5310	0.4619	0.2134

Value of Parameter M^e

Case of Radiative Equilibrium

Reflectivity Wall 1 = 0.9

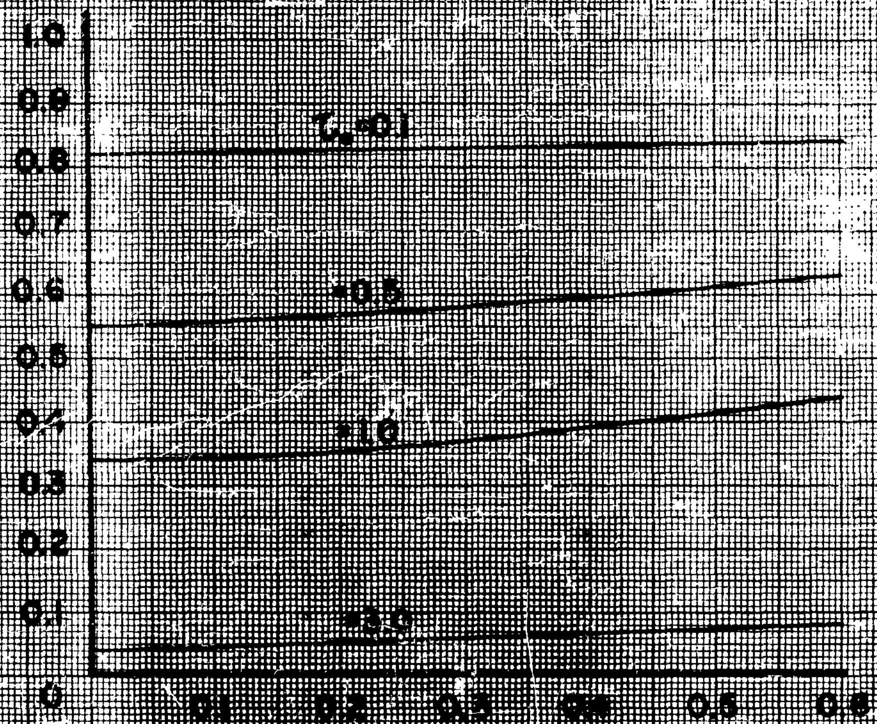
Optical Spacing	Reflectivity Wall 2		
	0.1	0.5	0.9
0.1	0.3079	0.2832	0.1646
0.2	0.3053	0.2811	0.1638
0.3	0.3029	0.2790	0.1631
0.4	0.3006	0.2770	0.1625
0.6	0.2961	0.2732	0.1612
0.8	0.2919	0.2696	0.1599
1.0	0.2878	0.2661	0.1587
1.5	0.2781	0.2578	0.1558
3.0	0.2529	0.2361	0.1477
5.0	0.2258	0.2122	0.1383

APPENDIX E

Results for case

of

Normal Incident Flux

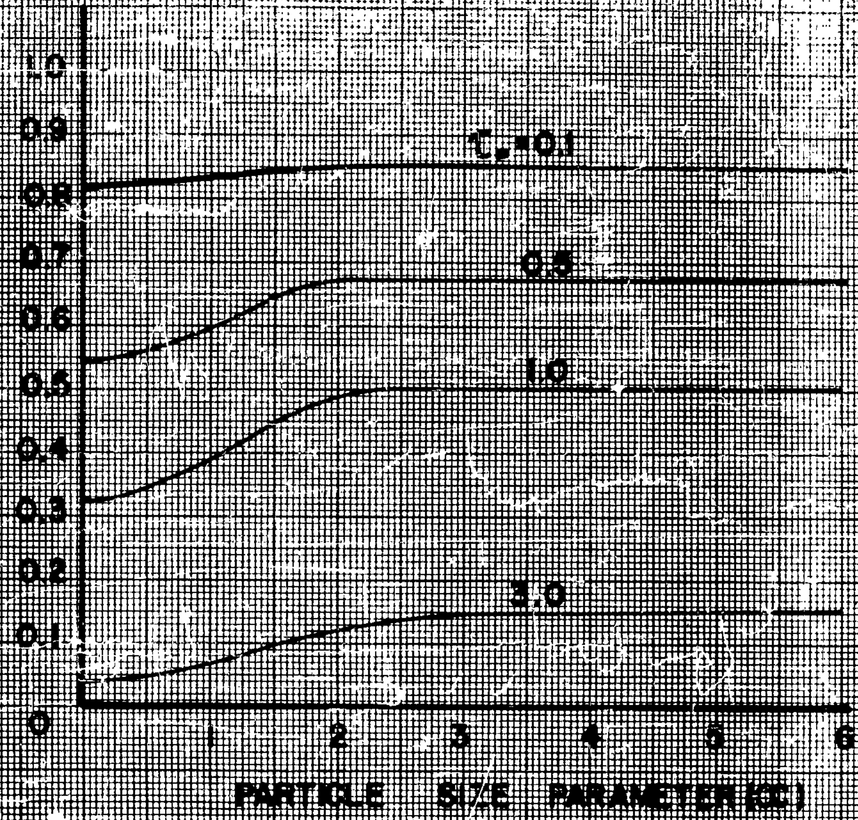


SCATTERING RATIO

PARAMETER

(NORMAL INCIDENT RADIATION)

FIGURE

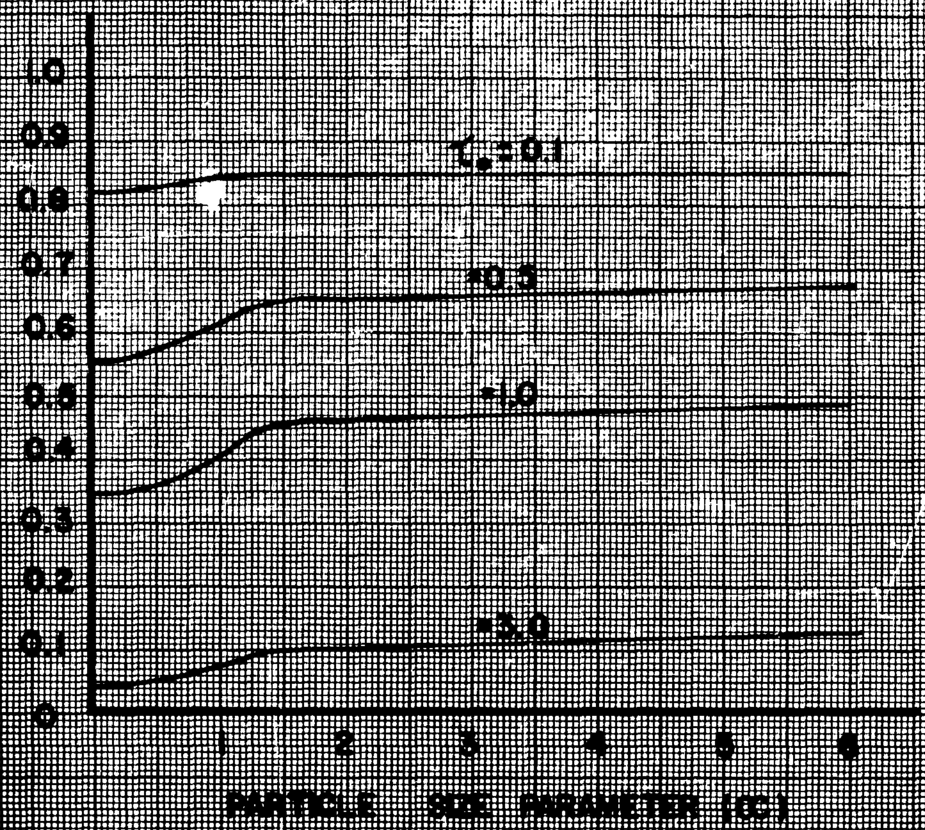


PARAMETER τ_o

(NORMAL INCIDENT RADIATION)

$m = 1.29 - 1.25i \quad \rho = 0.1$

FIGURE 5.0



PARAMETER Q^*

(NORMAL INCIDENT RADIATION)

$m = 2.0 - 0.6i$ $\rho = 0.1$

FIGURE 31

APPENDIX E

TABULATED RESULTS FOR THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.3

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8254	0.4682	0.8151
0.5	2.8201	1.5569	0.5480
1.0	2.8187	2.1919	0.3333
3.0	2.8182	2.7665	0.0454
5.0	2.8182	2.8131	0.0061

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5706	0.2610	0.4528
0.5	1.5689	0.8665	0.3047
1.0	1.5685	1.2199	0.1854
3.0	1.5683	1.5396	0.0252
5.0	1.5683	1.5655	0.0034

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3143	0.0535	0.0902
0.5	0.3142	0.1742	0.0607
1.0	0.3142	0.2447	0.0369
3.0	0.3142	0.3084	0.0050
5.0	0.3142	0.3136	0.0006

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8067	0.4272	0.8198
0.5	2.7734	1.4644	0.5606
1.0	2.7636	2.0980	0.3464
3.0	2.7603	2.7016	0.0492
5.0	2.7603	2.7542	0.0068

Reflectivity of Wall = 0.5

0.1	1.5648	0.2389	0.4568
0.5	1.5544	0.8211	0.3140
1.0	1.5513	1.1779	0.1943
3.0	1.5503	1.5173	0.0276
5.0	1.5503	1.5468	0.0038

Reflectivity of Wall = 0.9

0.1	0.3140	0.0492	0.0912
0.5	0.3136	0.1664	0.0630
1.0	0.3135	0.2384	0.0390
3.0	0.3134	0.3068	0.0055
5.0	0.3134	0.3128	0.0007

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 1.0

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7773	0.3429	0.8317
0.5	2.6952	1.2561	0.5963
1.0	2.6677	1.8798	0.3870
3.0	2.6568	2.5698	0.0638
5.0	2.6567	2.6460	0.0099

Reflectivity of Wall = 0.5

0.1	1.5556	0.1928	0.4656
0.5	1.5295	0.7133	0.3382
1.0	1.5206	1.0718	0.2204
3.0	1.5171	1.4674	0.0364
5.0	1.5171	1.5109	0.0056

Reflectivity of Wall = 0.9

0.1	0.3137	0.0402	0.0934
0.5	0.3126	0.1466	0.0687
1.0	0.3122	0.2205	0.0450
3.0	0.3121	0.3019	0.0074
5.0	0.3121	0.3108	0.0011

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 0.6

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8067	0.4272	0.8198
0.5	2.7734	1.4644	0.5606
1.0	2.7636	2.0980	0.3464
3.0	2.7603	2.7016	0.0492
5.0	2.7603	2.7542	0.0068

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5648	0.2389	0.4568
0.5	1.5544	0.8211	0.3140
1.0	1.5513	1.1779	0.1943
3.0	1.5503	1.5173	0.0276
5.0	1.5503	1.5468	0.0038

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3140	0.0492	0.0912
0.5	0.3136	0.1664	0.0630
1.0	0.3135	0.2384	0.0390
3.0	0.3134	0.3068	0.0055
5.0	0.3134	0.3128	0.0007

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 2.0

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7877	0.2719	0.8515
0.5	2.7282	1.0563	0.6687
1.0	2.7075	1.6613	0.4823
3.0	2.6975	2.5119	0.1150
5.0	2.6973	2.6620	0.0250

Reflectivity of Wall = 0.5

0.1	1.5588	0.1528	0.4759
0.5	1.5401	0.5968	0.3773
1.0	1.5335	0.9412	0.2730
3.0	1.5303	1.4250	0.0652
5.0	1.5302	1.5102	0.0141

Reflectivity of Wall = 0.9

0.1	0.3138	0.0321	0.0953
0.5	0.3130	0.1222	0.0763
1.0	0.3127	0.1925	0.0554
3.0	0.3126	0.2912	0.0132
5.0	0.3126	0.3085	0.0028

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 4.0

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8038	0.2505	0.8333
0.5	2.7716	0.9868	0.6226
1.0	2.7598	1.5768	0.4434
3.0	2.7532	2.4900	0.1265
5.0	2.7530	2.6897	0.0363

Reflectivity of Wall = 0.5

0.1	1.5639	0.1405	0.4645
0.5	1.5538	0.5537	0.3488
1.0	1.5501	0.8860	0.2489
3.0	1.5480	1.4001	0.0710
5.0	1.5479	1.5124	0.0204

Reflectivity of Wall = 0.9

0.1	0.3140	0.0296	0.0928
0.5	0.3136	0.1127	0.0700
1.0	0.3134	0.1798	0.0500
3.0	0.3133	0.2835	0.0143
5.0	0.3133	0.3062	0.0041

Refractive index of scattering medium = 1.25 - 1.25 i

Particle size parameter α = 6.0

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8108	0.2492	0.8513
0.5	2.7871	0.9795	0.6783
1.0	2.7774	1.5672	0.5080
3.0	2.7712	2.4897	0.1543
5.0	2.7709	2.6997	0.0453

Reflectivity of Wall = 0.5

0.1	1.5661	0.1396	0.4740
0.5	1.5587	0.5483	0.3791
1.0	1.5556	0.8782	0.2844
3.0	1.5537	1.3959	0.0865
5.0	1.5536	1.5137	0.0253

Reflectivity of Wall = 0.9

0.1	0.3141	0.0294	0.0946
0.5	0.3138	0.1114	0.0759
1.0	0.3137	0.1777	0.0570
3.0	0.3136	0.2819	0.0173
5.0	0.3136	0.3056	0.0051

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 0.6

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8007	0.4061	0.8225
0.5	2.7590	1.4149	0.5686
1.0	2.7464	2.0488	0.3552
3.0	2.7422	2.6767	0.0522
5.0	2.7422	2.7350	0.0074

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5629	0.2273	0.4587
0.5	1.5498	0.7952	0.3192
1.0	1.5459	1.1534	0.1998
3.0	1.5445	1.5076	0.0293
5.0	1.5445	1.5405	0.0042

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3140	0.0470	0.0917
0.5	0.3134	0.1615	0.0642
1.0	0.3133	0.2341	0.0403
3.0	0.3132	0.3058	0.0059
5.0	0.3132	0.3124	0.0008

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 1.20

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7704	0.2922	0.8419
0.5	2.6748	1.1170	0.6309
1.0	2.6397	1.7271	0.4296
3.0	2.6235	2.4971	0.0829
5.0	2.6233	2.6043	0.0148

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5534	0.1646	0.4718
0.5	1.5229	0.6364	0.3590
1.0	1.5115	0.9892	0.2459
3.0	1.5062	1.4336	0.0476
5.0	1.5061	1.4952	0.0085

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3136	0.0346	0.0948
0.5	0.3123	0.1314	0.0732
1.0	0.3118	0.2046	0.0505
3.0	0.3116	0.2967	0.0098
5.0	0.3116	0.3094	0.0017

THE CASE OF NORMAL INCIDENT RADIATION

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 2.40

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7710	0.2631	0.8462
0.5	2.6791	1.0292	0.6505
1.0	2.6231	1.6231	0.4599
3.0	2.6095	2.4437	0.1043
5.0	2.6089	2.5795	0.0218

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5536	0.1483	0.4742
0.5	1.5220	0.5868	0.3703
1.0	1.5087	0.9310	0.2635
3.0	1.5015	1.4062	0.0600
5.0	1.5013	1.4844	0.0125

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3136	0.0313	0.0952
0.5	0.3123	0.1213	0.0756
1.0	0.3117	0.1929	0.0542
3.0	0.3114	0.2917	0.0123
5.0	0.3114	0.3079	0.0025

Refractive index of scattering medium = 2.0 - 0.6 i

Particle size parameter α = 3.00

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7679	0.2706	0.8419
0.5	2.6617	1.0514	0.6351
1.0	2.6185	1.6480	0.4396
3.0	2.5962	2.4464	0.0937
5.0	2.5957	2.5705	0.0188

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5526	0.1526	0.4720
0.5	1.5187	0.6004	0.3621
1.0	1.5045	0.9472	0.2524
3.0	1.4971	1.4108	0.0540
5.0	1.4970	1.4825	0.0108

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3135	0.0322	0.0948
0.5	0.3121	0.1243	0.0741
1.0	0.3115	0.1967	0.0520
3.0	0.3112	0.2934	0.0111
5.0	0.3112	0.3082	0.0022

Isotropic Case

Scattering to extinction ratio = 0.065

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.8172	0.4502	0.8170
0.5	2.8001	1.5172	0.5527
1.0	2.7953	2.1524	0.3380
3.0	2.7938	2.7394	0.0466
5.0	2.7938	2.7884	0.0063

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5680	0.2513	0.4545
0.5	1.5627	0.8471	0.3083
1.0	1.5612	1.2023	0.1886
3.0	1.5608	1.5304	0.0260
5.0	1.5608	1.5577	0.0035

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3142	0.0516	0.0906
0.5	0.3139	0.1709	0.0616
1.0	0.3139	0.2421	0.0377
3.0	0.3139	0.3078	0.0052
5.0	0.3139	0.3133	0.0007

THE CASE OF NORMAL INCIDENT RADIATION

Isotropic Case

Scattering to extinction ratio = 0.170

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7973	0.4069	0.8216
0.5	2.7495	1.4171	0.5649
1.0	2.7352	2.0497	0.3506
3.0	2.7305	2.6683	0.0503
5.0	2.7304	2.7239	0.0070

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5619	0.2279	0.4585
0.5	1.5468	0.7976	0.3176
1.0	1.5423	1.1560	0.1976
3.0	1.5408	1.5057	0.0283
5.0	1.5408	1.5371	0.0039

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3139	0.0471	0.0917
0.5	0.3133	0.1623	0.0640
1.0	0.3131	0.2351	0.0399
3.0	0.3131	0.3060	0.0057
5.0	0.3131	0.3123	0.0008

Isotropic Case

Scattering to extinction ratio = 0.285

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7747	0.3574	0.8267
0.5	2.6881	1.2948	0.5798
1.0	2.6600	1.9192	0.3668
3.0	2.6498	2.5761	0.0555
5.0	2.6497	2.6414	0.0081

Reflectivity of Wall = 0.5

Optical Spacing	$-M^n$	N^n	Q^n
0.1	1.5548	0.2010	0.4630
0.5	1.5272	0.7361	0.3292
1.0	1.5181	1.0956	0.2092
3.0	1.5148	1.4727	0.0317
5.0	1.5148	1.5100	0.0046

Reflectivity of Wall = 0.9

Optical Spacing	$-M^n$	N^n	Q^n
0.1	0.3136	0.0419	0.0929
0.5	0.3125	0.1514	0.0670
1.0	0.3121	0.2256	0.0428
3.0	0.3121	0.2256	0.0428
5.0	0.3120	0.3110	0.0009

THE CASE OF NORMAL INCIDENT RADIATION

Isotropic Case

Scattering to extinction ratio = 0.425

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7460	0.2947	0.8331
0.5	2.6031	1.1245	0.6003
1.0	2.5511	1.7267	0.3909
3.0	2.5292	2.4345	0.0646
5.0	2.5289	2.5170	0.0101

Reflectivity of Wall = 0.5

0.1	1.5457	0.1666	0.4687
0.5	1.4994	0.6482	0.3456
1.0	1.4820	1.0034	0.2269
3.0	1.4746	1.4194	0.0376
5.0	1.4745	1.4676	0.0058

Reflectivity of Wall = 0.9

0.1	0.3133	0.0351	0.0945
0.5	0.3113	0.1354	0.0714
1.0	0.3106	0.2107	0.0473
3.0	0.3102	0.2987	0.0078
5.0	0.3102	0.3088	0.0012

Isotropic Case

Scattering to extinction ratio = 0.60

Reflectivity of Wall = 0.1

Optical Spacing	$-M^n$	N^n	Q^n
0.1	2.7080	0.2117	0.8421
0.5	2.4768	0.8692	0.6321
1.0	2.3773	1.4117	0.4318
3.0	2.3245	2.1821	0.0844
5.0	2.3234	2.3012	0.0152

Reflectivity of Wall = 0.5

0.1	1.5536	0.1206	0.4766
0.5	1.4567	0.5117	0.3715
1.0	1.4217	0.8445	0.2581
3.0	1.4026	1.3168	0.0509
5.0	1.4022	1.3888	0.0092

Reflectivity of Wall = 0.9

0.1	0.3128	0.0260	0.0967
0.5	0.3094	0.1097	0.0785
1.0	0.3078	0.1835	0.0556
3.0	0.3069	0.2882	0.0110
5.0	0.3069	0.3040	0.0020

APPENDIX F

TABLE OF DOUBLE GAUSSIAN QUADRATURE WEIGHT FACTORS AND ORDINATES

$$\int_0^1 f(\mu)\, d\mu = \sum_{j=1}^{n} a_j f(\mu_j)$$

$n = 3$	$\mu_1 = 0.11270167$	$a_1 = 0.27777777$
	$\mu_2 = 0.50000000$	$a_2 = 0.44444444$
	$\mu_3 = 0.88729833$	$a_3 = 0.27777777$
$n = 4$	$\mu_1 = 0.06943184$	$a_1 = 0.1739274$
	$\mu_2 = 0.33000984$	$a_2 = 0.3260726$
	$\mu_3 = 0.66999052$	$a_3 = 0.3260726$
	$\mu_4 = 0.93056816$	$a_4 = 0.1739274$

TABLE OF REIZ QUADRATURE WEIGHT FACTORS AND ORDINATES

$$\int_0^{\infty} e^{-x} f(x)\, dx = \sum_{j=1}^{n} a_j f(x_j)$$

$x_1 = 0.2635603$	$a_1 = 0.5217556$
$x_2 = 1.4134030$	$a_2 = 0.3986668$
$x_3 = 3.5964258$	$a_3 = 0.0759424$
$x_4 = 7.0858102$	$a_4 = 0.0036118$
$x_5 = 12.6408007$	$a_5 = 0.0000234$

APPENDIX G

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

FOURTH ORDER APPROXIMATION

n	$P_n(+\mu_1,+\mu_1)$	$P_n(+\mu_1,+\mu_2)$	$P_n(+\mu_1,+\mu_3)$	$P_n(+\mu_1,+\mu_4)$	$P_n(+\mu_2,+\mu_2)$
1	0.038761	0.023255	0.045400	0.063588	0.107556
2	0.220988	0.199411	-0.065162	-0.387656	0.143176
3	0.059731	0.048089	0.034029	-0.060413	0.179949
4	0.150032	0.033951	-0.159232	0.134481	0.019005
5	0.034480	0.050819	-0.040444	0.017374	0.146887
6	0.107330	-0.044972	-0.006154	0.016999	0.047462
7	0.059948	0.040391	-0.042605	0.035834	0.053052
8	0.075820	-0.060483	0.073647	-0.080638	0.097367
9	0.042831	0.006488	0.027015	-0.067243	0.016409
10	0.065426	-0.054905	0.020472	0.075933	0.081848
11	0.055473	-0.024297	0.052185	0.059740	0.040507
12	0.047414	-0.029446	-0.037404	-0.037713	0.35988
13	0.043940	-0.042573	-0.018672	-0.019878	0.072951
14	0.051449	-0.000667	-0.020953	0.000913	0.011651
15	0.050350	-0.052197	-0.052209	-0.027580	0.061620
16	0.035152	0.011709	0.019504	0.014996	0.046090
17	0.049456	-0.030587	0.009322	0.055506	0.029387
18	0.039695	0.004106	0.011545	-0.010841	0.069106

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

FOURTH ORDER APPROXIMATION

n	$P_n(+\mu_2,+\mu_3)$	$P_n(+\mu_2,+\mu_4)$	$P_n(+\mu_3,+\mu_3)$	$P_n(+\mu_3,+\mu_4)$	$P_n(+\mu_4,+\mu_4)$
1	0.215914	0.309442	0.449737	0.624248	0.866008
2	-0.036115	-0.262731	0.041540	0.143166	0.639063
3	0.116480	-0.245018	0.087965	-0.145551	0.385792
4	-0.007183	0.004442	0.204823	-0.160414	0.173903
5	-0.083549	0.052016	0.113232	-0.051438	0.048627
6	-0.003317	-0.010693	0.021472	-0.004733	0.019370
7	-0.057370	0.038104	0.085332	-0.059473	0.061213
9	-0.083271	0.097077	0.130711	-0.114506	0.129469
9	-0.018387	0.010570	0.061373	-0.079083	0.180221
10	-0.026185	-0.097177	0.026876	0.020390	0.187792
11	-0.022466	-0.062354	0.080314	0.085816	0.152207
12	0.020984	0.027889	0.090313	0.068065	0.094626
13	0.013846	0.031863	0.035896	0.013815	0.044188
14	0.010607	0.000104	0.032561	-0.004030	0.023103
15	0.030843	0.018689	0.076966	0.025041	0.036935
16	0.026968	0.035274	0.069017	0.047565	0.073905
17	0.020718	-0.018074	0.030497	0.021449	0.112506
18	0.021504	-0.058432	0.045253	-0.031660	0.132926

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

FOURTH ORDER APPROXIMATION

n	$P_n(+\mu_1,-\mu_1)$	$P_n(+\mu_1,-\mu_2)$	$P_n(+\mu_1,-\mu_3)$	$P_n(+\mu_1,-\mu_4)$	$P_n(+\mu_2,-\mu_2)$
1	-0.038761	-0.023255	-0.045400	-0.063588	-0.107556
2	0.220989	0.199411	-0.065162	-0.387656	0.143176
3	-0.059731	-0.048089	-0.034029	0.060413	-0.179949
4	0.150032	0.033951	-0.159232	0.134481	0.019005
5	-0.034480	-0.050819	0.040444	-0.017374	-0.146867
6	0.107330	-0.044972	-0.006154	0.016999	0.047462
7	-0.059948	-0.040391	0.042605	-0.035834	-0.053052
8	0.075820	-0.060483	0.073647	-0.080638	0.097367
9	-0.042831	-0.006488	-0.027015	0.067243	-0.016409
10	0.065426	-0.054905	0.020472	0.075933	0.081848
11	-0.055473	0.024297	-0.052185	-0.059740	-0.040507
12	0.047414	-0.029446	-0.037404	-0.037713	0.035988
13	-0.043940	0.043573	0.018672	0.019878	-0.072951
14	0.051449	-0.000667	-0.020953	0.000913	0.011651
15	-0.050350	0.052197	0.052209	0.027580	-0.061620
16	0.035152	0.011709	0.019504	0.014496	0.046090
17	-0.049456	0.030587	-0.009322	-0.044406	-0.029387
18	0.039695	0.004106	0.011545	-0.010841	0.069106

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

FOURTH ORDER APPROXIMATION

n	$P_n(+\mu_2,-\mu_3)$	$P_n(+\mu_2,-\mu_4)$	$P_n(+\mu_3,-\mu_3)$	$P_n(+\mu_3,-\mu_4)$	$P_n(+\mu_4,-\mu_4)$
1	-0.215914	-0.309442	-0.449737	-0.624248	-0.866008
2	-0.036115	-0.262732	0.041540	0.143166	0.639068
3	-0.116481	0.245018	-0.087965	0.145551	-0.385742
4	-0.007183	0.004442	0.204823	-0.160414	0.173903
5	0.083549	-0.052016	-0.113233	0.051438	-0.048627
6	-0.003317	-0.010693	0.021472	-0.004733	0.019370
7	0.057370	-0.038104	-0.085332	0.059473	-0.061213
8	-0.083271	0.097077	0.130711	-0.114506	0.129469
9	0.018387	-0.010570	-0.061373	0.079083	-0.180221
10	-0.026185	-0.097177	0.026876	0.020390	0.187792
11	0.022466	0.062354	-0.080314	-0.085816	-0.152207
12	0.020984	0.027889	0.090313	0.068065	0.094626
13	-0.013846	-0.031863	-0.035896	-0.013815	-0.044188
14	0.010607	0.000104	0.032561	-0.004030	0.023103
15	-0.030843	-0.018689	-0.076966	-0.025041	-0.036935
16	0.026968	0.035274	0.069017	0.047565	0.073905
17	-0.020718	0.018074	-0.030497	-0.021449	-0.112506
18	-0.020718	0.018074	-0.030497	-0.021449	-0.112506
19	0.021504	-0.058432	0.045253	-0.031660	0.132926

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

THIRD ORDER APPROXIMATION

n	$P_n(+\mu_1,+\mu_1)$	$P_n(+\mu_1,+\mu_2)$	$P_n(+\mu_1,+\mu_3)$	$P_n(+\mu_2,+\mu_2)$
1	0.012430	0.057526	0.010484	0.251265
2	0.266952	0.089100	-0.333888	+0.033919
3	0.028700	0.082889	-0.008594	+0.217847
4	0.147190	-0.090890	0.038754	0.083541
5	0.048271	0.029010	0.000424	+0.029545
6	0.092940	-0.084619	0.105489	0.131717
7	0.057301	-0.050849	0.006547	0.069009
8	0.057095	-0.017890	-0.112750	0.028202
9	0.071605	-0.075071	-0.006786	0.105107
10	0.036998	-0.015033	0.036991	0.059938
11	0.076036	-0.025047	0.000959	0.022266
12	0.022929	0.005988	0.043642	0.080149
13	0.076021	0.036489	0.003814	0.047889
14	0.018036	-0.006827	-0.072027	0.017806
15	0.078348	0.051928	-0.005054	0.067977
16	0.013998	0.018495	0.040848	0.045913
17	0.068120	0.019566	-0.001276	0.020707
18	0.022336	0.027815	0.014192	0.063792

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

THIRD ORDER APPROXIMATION

n	$P_n(+\mu_2,+\mu_3)$	$P_n(+\mu_3,+\mu_3)$	$P_n(+\mu_1-\mu_1)$	$P_n(+\mu_1,-\mu_2)$
1	0.443688	0.787732	-0.012430	-0.057526
2	-0.078726	0.467298	0.266952	0.089100
3	-0.167301	0.183989	-0.028700	-0.082889
4	-0.027595	0.040286	0.147190	-0.090890
5	-0.013142	0.045059	-0.048271	-0.029010
6	-0.109504	0.125353	0.092940	-0.084619
7	-0.097971	0.186636	-0.057301	0.050849
8	0.023996	0.176521	0.057095	-0.017890
9	0.077130	0.109496	-0.071605	0.075071
10	0.026973	0.041831	-0.036998	-0.015033
11	0.004518	0.021704	-0.076036	0.025047
12	0.047253	0.053840	0.022929	0.005988
13	0.052216	0.114600	-0.076021	-0.036489
14	-0.014125	0.122145	0.018036	-0.006827
15	-0.056721	0.102970	-0.078348	-0.051928
16	-0.028649	0.060283	0.013998	0.018495
17	-0.001947	0.032293	-0.068120	-0.019566
18	-0.021534	0.040021	0.022336	0.027815

TABLE OF INTEGRATED LEGENDRE POLYNOMIALS

THIRD ORDER APPROXIMATIONS

n	$P_n(+\mu_1,-\mu_3)$	$P_n(+\mu_2,-\mu_2)$	$P_n(+\mu_2,-\mu_3)$	$P_n(+\mu_3,-\mu_3)$
1	-0.010484	-0.251265	-0.443688	-0.787732
2	-0.333888	0.033919	-0.078726	0.467298
3	0.008594	-0.217847	0.167301	-0.183989
4	0.038754	0.083541	-0.027595	0.040286
5	-0.000424	-0.029545	0.013142	-0.045059
6	0.105489	0.131717	-0.109504	0.125353
7	-0.006547	-0.069009	0.097971	-0.186636
8	-0.112750	0.028202	0.023996	0.176521
9	0.006786	-0.105107	-0.077130	-0.109496
10	0.036991	0.059938	0.026973	0.041831
11	-0.000959	-0.022266	-0.004518	-0.021704
12	0.043642	0.080149	0.047253	0.053840
13	-0.003814	-0.047889	-0.052216	-0.114600
14	-0.072027	0.017806	-0.014125	0.122145
15	0.005054	-0.067977	0.056721	-0.102970
16	0.040848	0.045913	-0.028649	0.060283
17	0.001276	-0.020707	0.001947	-0.032293
18	0.014192	0.063792	-0.021534	0.040021

APPENDIX H

LIST OF SYMBOLS

Alphabetic Symbol	Meaning	Dimensional Units
A_j	Adjusted quadrature weight factor	$(Btu)(hr)^{-1}(ft)^{-2}(R)^{-4}$
a_j	Quadrature weight factor	none
a_n	Coefficient of Lengendre expansion	none
b	Computed term of matrix	none
C	Constant	none
c	Velocity of light	$(ft)(hr)^{-1}$
D	Particle diameter	(ft)
D	Computed parameter, equation (36)	none
d	Matrix coefficient	none
E	Computed parameter, equation (34)	none
e	Base of natural logarithm	none
e	Matrix coefficient	none
F	Computed parameter, equation (34)	none
	Monochromatic radiation flux	$(Btu)(ft)^{-2}$
G	Computed parameter, equation (34)	none
H	Computed parameter, equation (34)	none
h	Planck's constant	$(Btu)(hr)^{-1}$
I	Monochromatic intensity of radiation	$(Btu)(ft)^{-2}(stearad)^{-1}$
K^e	Extinction cross section	none
K	Computed parameter, equation (34)	none
k	Boltzman's constant	$(Btu)(R)^{-1}$
L	Computed parameter, equation (34)	none
M	Computed parameter, equation (36)	none
m	Complex refractive index	none
m	Scattering mass	(lb_m)
N	Computed parameter, equation (36)	none
P_n	Legendre polynomial of order n	none
Q	Computed parameter, equation (36)	none
q_{net}	Net radiant flux	$(Btu)(hr)^{-1}(ft)^{-2}$
R	Monochromatic radiosity	$(Btu)(ft)^{-2}$
S	Scattering function	none
s	Distance along a ray	(ft)
T	Temperature	(R)
x	Normal coordinate distance	(ft)
$x_{i,\alpha}$	i^{th} eigenvector associated with the α^{th} eigenvalue of a matrix	none

Greek Symbol	Meaning	Dimensional Units
α	Particle size parameter	none
β	Monochromatic mass extinction coefficient	$(ft)^2(lb_m)^{-1}$
γ	Eigenvalue of a matrix	none
δ	Kronecker delta	none
ϵ	Monochromatic surface emissivity	none
θ	Polar angle	radians
κ	Monochromatic mass absorption coefficient	$(ft)^2(lb_m)^{-1}$
λ	Radiation wave length	(ft)
μ	Cosine θ	none
ν	Frequency of Radiation	$(hr)^{-1}$
ξ	Computed parameter, equation (77)	none
π	3.1416	none
ρ	Mass density	$(lb_m)(ft)^{-3}$
ρ	Monochromatic surface reflectivity	none
σ	Monochromatic mass scattering coefficient	$(ft)^2(lb_m)^{-1}$
τ	Optical depth	none
φ	Azimuthal angle	radians
φ_i	Computed parameter, equation (83)	none
Ω	Chromey extinction parameter	none
ω	Solid angle	stearadian

SUBSCRIPTS

α	Iteration index
bb	Refers to black body
i	Iteration index
j	Iteration index
a	Refers to atmosphere
p	Iteration index
q	Iteration index

SUPERSCRIPTS

e	Equilibrium case
n	Normal incidence case
o	Refers to distant source

Aeronautical Research Laboratories, Wright-Patterson AFB, Ohio. AN INVESTIGATION OF RADIANT HEAT TRANSFER IN ABSORBING, EMITTING AND SCATTERING MEDIA by Tom J. Love, Jr., U. of Oklahoma Rsch. Institute, Norman, Okla. January 1963. 303 p. incl. illus. tables. (Project 7116;Task 7116-03) (Contract AF 33(657)-8859) (ARL 63-3) Unclassified Report

This is a study designed to examine the effect of anisotropic scattering on radiant heat transfer. The equation of transfer for a scattering, absorbing, and emitting medium is simplified by restricting the analysis to anially symmetric, plane parallel geometry. The scattering mediu are assumed composed of spherical particles of uniform diameter and

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complex refractive index. The scattering functions are taken from the literature and are the results of application of the Mie theory of electromagnetic scattering.

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